EFFECTS OF THE REST MASS OF THE NEUTRINO (ANTINEUTRINO) ON THE
SCATTERING OF $\nu (\bar{\nu})$ BY $^{6}\text{Cl}^{12}$

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Expressions are obtained for the differential cross sections of the processes $\nu + ^{6}\text{Cl}^{12} \rightarrow ^{7}\text{N}^{12} + e^{-}$ and $\bar{\nu} + ^{6}\text{Cl}^{12} \rightarrow ^{7}\text{B}^{12} + e^{+}$ for the shell model of the nucleus and the harmonic oscillator model. We analyze the effect of the rest mass of the neutrino (antineutrino) on the degree of longitudinal polarization of the electrons (positrons), the angular electron-neutrino (positron-antineutrino) correlation coefficient, and the charge asymmetry.

The Grand Unification models of the fundamental interactions of elementary particles (see [1], for example) predict a nonzero mass for the neutrino. There is also experimental evidence for the existence of a nonzero rest mass for the electron antineutrino ($25 \text{eV} < m(\nu_{e}) < 46 \text{eV}$) [2]. In addition, a recent study of $\beta$-decay of tritium indicates the possible existence of still another kind of heavy electron neutrino with a mass of 17.1 keV [3]. Hence it is important to consider effects due to a nonzero rest mass of $\nu(\bar{\nu})$ in different electroweak interaction processes.

In the present paper, which is a continuation and extension of [4-6], we calculate the differential cross sections of the processes

\begin{align*}
\nu + ^{6}\text{Cl}^{12} & \rightarrow ^{7}\text{N}^{12} + e^{-}, \\
\bar{\nu} + ^{6}\text{Cl}^{12} & \rightarrow ^{7}\text{B}^{12} + e^{+}
\end{align*}

for longitudinal polarizations of the leptons, and also in the case of unpolarized leptons. The structure of the nucleus is described in terms of the multipole expansion of the weak nucleon current [7].

In the low-energy current-current theory of the weak interaction ($q_{\alpha}^{2} \ll m_{W}^{2}$) the matrix element of the transition $|i> \rightarrow |f>$ has the form

$$
<f|\hat{H}_{W}|i> = -\frac{G_{F}}{\sqrt{2}} l_{s} f_{s}^{e},
$$

where $G_{F} \approx 10^{-5} m_{p}^{-2}$ is the Fermi constant of the weak interaction;

$$
l_{s} = i\bar{u}_{s}({\alpha}_{V} + {\alpha}_{A} {\gamma}_{5}) u_{i},
$$

is the electron current, $\alpha_{V}$ and $\alpha_{A}$ are the coupling constants of its vector and axial-vector components, $u_{j}$ ($j = 1, 2$) are the Dirac spinor amplitudes;

$$
f_{s}^{e} = \int d r e^{-iqr} <f|\hat{J}_{s}^{e}(r)|i>.
$$
is the Fourier transform of the nucleon current matrix element \( \hat{J}_*^\alpha h(r) \), whose isospin structure is given by \([7, 8]\)

\[
\hat{J}_*^\alpha (r) = [\hat{p}_{\alpha}^V]_*^\alpha (r) + \hat{p}_{\alpha}^A (r) i_{MR}.
\]

Here \( \hat{p}_V(T) \) and \( \hat{p}_A(T) \) are the coupling constants of the vector \( \hat{I}_\alpha (r) \) and axial-vector \( \hat{A}_\alpha (r) \) parts (isoscalars for \( T = 0 \) and isovectors for \( T = 1 \)) of the nucleon current (in the reactions considered here the weak nucleon current is an isovector \( (T = 1) \) and is charged \( (M_T = +1) \)); \( \delta_q = (p_v - p_e)_\alpha \) is the transferred four-momentum; \( p_{v\alpha} \) and \( p_{e\alpha} \) are the four-momenta of the \( v(v) \) and the \( e^-(e^+) \).

The differential cross sections of the reactions (1) and (2), calculated on the basis of (3), to lowest order of perturbation theory in the constant \( G_F \), are given by

\[
\left( \frac{d\sigma}{d\Omega} \right)_i = \frac{G_F^2}{12\pi} \frac{a_{1i} p_{e\alpha} E_e}{\sqrt{1 - m_e^2/E_e^2}} F(f_i), \quad i = 1, ..., 5,
\]

where

\[
F(f_i) = f_i(\Phi_1 + f_2 \Phi_2 + Re(f_3 \Phi_3) + f_4 \Phi_4 + f_5 \Phi_5).
\]

Here \( p_e = \sqrt{E_e^2 - m_e^2} \), \( E_e = E_v - \Delta E \) and \( m_e \) are the momentum, energy, and rest mass of the electron (positron); \( p_v, E_v, \) and \( m_v \) are the momentum, energy, and rest mass of the neutrino (antineutrino); \( \Delta E = E_v - E_e = E_f - E_i \) is the energy difference of the transition. The functions \( f_i \) \((i = 1, ..., 5)\) take into account the structure of the nucleus. In the special case of transitions (1) and (2) between ground states and without the inclusion of currents of the second kind \([9]\), it turns out that two of the functions vanish

\[
\Phi_1 = \Phi_5 = 0,
\]

and the remaining three functions \( \Phi_2, \Phi_3 \), and \( \Phi_4 \) are given by

\[
\begin{align*}
\Phi_2 &= |\hat{p}_{\alpha}^{(1)}| < 1 + 1 1 : L_{1;1}^{11} : 0^+ 0 > |^2, \\
\Phi_3 &= |\hat{p}_{\alpha}^{(1)}| < 1 + 1 1 : T_{1;1}^{11} : 0^+ 0 > |^2, \\
\Phi_4 &= \hat{p}_{\alpha}^{(1)} | L_{1;1}^{11} : 0^+ 0 > * < 1 + 1 1 : T_{1;1}^{11} : 0^+ 0 > |^2.
\end{align*}
\]

These expressions involve the vector transverse magnetic \( (\hat{T}_{1;1}^{11} {mag}) \), axial-vector longitudinal-spin \( (\hat{T}_{1;1}^{11} {els}) \), and transverse electric \( (\hat{T}_{1;1}^{11} {els}) \) multipole operators of the nucleus; the symbol \( :: \) denotes a double reduced matrix element of the multipole operators of the nucleus, i.e., a matrix element reduced simultaneously in the angular momentum and isospin bases. The functions \( f_i \) \((i = 1, ..., 5)\) in (8) for the general case of arbitrary polarizations of the \( v(v) \) and \( e^-(e^+) \) are complicated, and therefore we do not present them here.

In the case of longitudinal polarization of \( v(v) \) and \( e^-(e^+) \) the differential cross sections of the reactions (1) and (2) are given by (7) through (9), in which the functions \( f_2, f_3, \) and \( f_5 \) are equal to

\[
\begin{align*}
f_2 &= \eta_1 \left[ 1 - s_v(p_v \cdot p_e) - 2(q \cdot p_v)(q \cdot p_e) \right], \\
f_3 &= \eta_1 \left[ 1 - s_e(q \cdot p_e) \right], \\
f_5 &= 2\eta_1 q^0 (p_e^\alpha - s_e^\alpha),
\end{align*}
\]

Here and below the upper (lower) sign refers to process (1) (process (2)); \( s_v = \pm 1 \) and \( s_e = \pm 1 \) are the helicity and velocity of the \( v(v) \); \( s_e = \pm 1 \) and \( s_e = \| \cdot \| = p_e/E_e \) are the helicity and velocity of the \( e^-(e^+) \), \( m_1 = p_1/E_1 \) and \( m_1 = \| \cdot \| = p_1/E_1 \) are the momentum and energy of the \( e^-(e^+) \), and \( q^0 = q/q \) is a unit vector.

Averaging (6) and (10) over the spin states of the \( v(v) \) and summing over the spin states of the \( e^-(e^+) \), we obtain the following expressions for the functions \( f_2, f_3, \), and \( f_5 \):

\[
\begin{align*}
f_2 &= (1 + x^2)[1 - (s_v^\alpha s_e^\alpha) + 2(q \cdot p_v)(q \cdot p_e)] - \eta_1, \\
f_3 &= \pm 4x q^0 (p_e - s_e^\alpha) - \eta_1, \\
f_5 &= (1 + x^2)[1 - (q \cdot p_v)(q \cdot p_e)] - \eta_1.
\end{align*}
\]

For further analysis we used the shell model of the nucleus. In the ground state \( C^{12} \) \((0^+ 0)\) the shells \( 1s_{1/2} - 2p_{1/2} \) are completely filled. The excited state \( C^{12}(1 + 1; \Delta E = 15.11 \text{ MeV}) \) or the ground states \( N^{12}(1+1) \) and \( B^{12}(1+1) \) are \( 1p-1h \) states (particle-hole states) and include the \( 1p_{1/2} \) and \( 2s_{1/2} - 1d_{5/2} \) levels. Any of the matrix elements

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