The motion of solid particles in a fluid flow is represented as a random process with independent increments. The resulting kinetic equation for the particle distribution has the form previously proposed [1]. The solution to this equation provides a system of equations for the hydrodynamics of the assembly of solid particles. These equations differ from ones previously proposed [2, 3] in having additional terms related to relative motion of the components, whose presence is due to anisotropy in the distribution of the normal stresses in the pseudogas.

§1. Derivation of the kinetic equation. Consider the motion of a large set of particles in a flow of fluid. The speed of a particular particle is influenced by forces of three types: external mass forces, interaction between the particle and the flow, and collisions.

Each particle perturbs the carrier and thus affects the interactions of the other particles, so the motion of one particle will, in general, be dependent on the motion of all.

The numerous random factors will result in a relatively smooth and slow variation in the speed of the flow. Direct collisions between particles will also affect the velocities, but such interactions are essentially short-range ones, whereas the others are not.

If the fluid is of low viscosity, particle collisions may be considered as statistically independent.

Let \( \{u, x\} \) be the radius vector of the point representing the state of a system of \( N \) particles in phase space:

\[
\{u, x\} = \{u(1), u(2), \ldots, u(N); x(1), x(2), \ldots, x(N)\}.
\]

Here \( u(i) \) is the velocity vector of particle \( i \), whose center of mass has radius vector \( x(i) \) relative to a fixed Cartesian coordinate system.

The following assumptions are made:

1. The velocity vector \( u(t) \) of the point in phase space may be considered as a random variable with independent increments.

2. Particle collisions may be represented as collisions of elastic spheres.

3. We can neglect simultaneous collisions of three or more particles.

The last assumption is largely a consequence of the previous one and can be taken as applying to any short-range interaction and any number of particles, even when close packing is approached [4].

A random process with independent increments can be represented as the sum of two random processes: a continuous diffusion and a random process composed of steps in the initial process. To determine the conditional probability density \( \varphi(t, x, u) \) for transition from one state to another we have [5]

\[
\frac{\partial \varphi}{\partial t} = - \sum_{i=1}^{N} \sum_{a=1}^{3} \frac{\partial}{\partial u_a(i)} \left[ a_s(i) \frac{\partial \varphi}{\partial u_a(i)} + \sum_{b=1}^{3} B_{sb}(i) \frac{\partial \varphi}{\partial u_b(b)} \right] + \sum_{1 \leq i < j < N} \left[ \int_{0}^{t} \varphi(t', x, u) \right] \frac{\partial A_{ij}(i)}{\partial u(i)} + \frac{\partial}{\partial u(i)} \left[ (u(i) - u(i - 1), u(i) + \sum_{a=1}^{3} l_a \left( u_a(j) - u_a(i), u(j+1), \ldots, u(j+1), \ldots, u(N) - \sum_{a=1}^{3} l_a \left( u_a(j) - u_a(i), \right) \right) \right]
\]
\[
\mathbf{u}_s^{(2)} = \mathbf{u}^{(3)} - \sum_{s=1}^{3} l_s (\mathbf{u}_s^{(3)} - \mathbf{u}_s^{(1)}). 
\]  
(1.4)

We now apply the hypothesis of molecular chaos:
\[
g(t; \mathbf{x}^{(1)}, \mathbf{u}^{(1)}; \mathbf{x}^{(2)}, \mathbf{u}^{(2)}) = \int f(\mathbf{x}^{(1)}, \mathbf{u}^{(1)}; t) f(\mathbf{x}^{(2)}, \mathbf{u}^{(2)}; t) \, dt. 
\]  
(1.5)

This applies for dense suspensions within this collision model [4].

Then (1.3) and (1.5) give an equation for \( f \):
\[
\frac{\partial f}{\partial t} + \sum_{s=1}^{3} \mathbf{u}_s \frac{\partial f}{\partial \mathbf{u}_s} = \sum_{\beta=1}^{3} B_{s\beta} \frac{\partial f}{\partial \mathbf{u}_\beta} + \delta^2 \int [x^f f' - xf f] \, k d\mu, 
\]  
(1.6)

in which the symbols in the second term on the right-hand side are standard ones in the kinetic theory of gases.

Equation (1.6) coincides with the equation previously proposed [1].

2. Characterization of the continuous component.

The equation of motion of a particle may be put as
\[
\frac{d\mathbf{u}}{dt} = \frac{1}{m} (\mathbf{G} + \mathbf{F} + \mathbf{K}), 
\]  
(2.1)

in which \( \mathbf{G} \) is the external mass force (e.g., gravity), \( \mathbf{F} \) is the force from the interaction of the particles with the fluid, and \( \mathbf{K} \) is the force between colliding particles, in which \( \mathbf{K}(t) \) may be represented as a sum of \( \delta \)-functions.

The characteristic time of free motion between two collisions is much less than the characteristic time for \( \mathbf{F}(t) \), so the characteristics of the diffusion operator in (1.6) may be determined from a simplified form of (2.1):
\[
\frac{d\mathbf{u}}{dt} = m^{-2} (\mathbf{G} + \mathbf{F}). 
\]  
(2.2)

In what follows we consider only the case in which the particles are much more dense than the fluid. Then for the force exerted on a particle by the fluid we have
\[
\mathbf{F}/m = \Phi(\mathbf{e}, |\mathbf{s} - \mathbf{u}|)(\mathbf{s} - \mathbf{u}), 
\]  
(2.3)

in which \( \mathbf{s} \) is the velocity of the fluid, \( \mathbf{u} \) is the velocity of the particle, and \( \mathbf{e} \) is the mean relative volume of the fluid in a sufficiently large space around a particle, which is related to the number of particles in that volume by \( \mathbf{e} = 1 - n v_0 \), in which \( v_0 \) is the volume of a particle.

Estimates [9] show that only fairly large inhomogeneities play a major part in energy transfer from gas to particles.

A semiempirical analysis [7] indicates that we must allow for the dependence of \( \mathbf{F} \) on \( \mathbf{e} \); this agrees well with experiment.

We represent \( \mathbf{s} \) and \( \mathbf{u} \) as
\[
\mathbf{s} = \mathbf{q} + \mathbf{w}, \quad \mathbf{u} = \mathbf{w} + \mathbf{v}, \quad \mathbf{w} = \int u f d\mu, \quad \mathbf{v} = \int v f d\mu = 0, \quad \mathbf{w}(t) d\xi = 0, \quad \mathbf{v}(t) d\xi = 0, \]  
(2.4)
in which \( \mathbf{q} \) is the mean velocity of the fluid near a particle when the mean porosity of the suspension in that region is \( \mathbf{e} \).

We further assume that \( |\mathbf{w}| \) and \( |\mathbf{v}| \) are small relative to \( |\mathbf{q} - \mathbf{w}| \), and put \( \mathbf{F} \) as
\[
\frac{\mathbf{F}}{m} = \Phi(\mathbf{e}, |\mathbf{q} - \mathbf{w}|)(\mathbf{q} - \mathbf{w}) + \Phi(\mathbf{e}, |\mathbf{q} - \mathbf{w}|)(\mathbf{w} - \mathbf{v}) + \frac{\partial \Phi(\mathbf{e}, |\mathbf{q} - \mathbf{w}|)}{\partial \mathbf{e}} \Delta \mathbf{e}(\mathbf{q} - \mathbf{w}), 
\]  
(2.5)

neglecting terms of order \( |\mathbf{w} - \mathbf{v}|^2, (\Delta \mathbf{e})^2 \), etc., where \( \Delta \mathbf{e} \) is the fluctuation in the porosity around a particle.

Expression (2.4) can be put as
\[
m^{-2} \Phi_e = - a_*^* + A_{a*} (a = 1, 2, 3), 
\]  
(2.6)

in which \( a_* = - \Phi(\mathbf{e}, |\mathbf{q} - \mathbf{w}|)(\mathbf{q} - \mathbf{w}) + \Phi(\mathbf{e}, |\mathbf{q} - \mathbf{w}|) \mathbf{w}, 
\]

\( A_{a*} = \Phi(\mathbf{e}, |\mathbf{q} - \mathbf{w}|) \mathbf{w} + \frac{\partial \Phi(\mathbf{e}, |\mathbf{q} - \mathbf{w}|)}{\partial \mathbf{e}} \Delta \mathbf{e}(\mathbf{q} - \mathbf{w}). \)  

The time-average of \( a_{a*} \) is zero, so we have
\[
\frac{d\mathbf{v}}{dt} = \frac{\mathbf{G}}{m} + \frac{\mathbf{d}}{dt} - a_*^* + A. \]  
(2.7)

For the steady state
\[
\mathbf{G}/m + \Phi(\mathbf{e}, |\mathbf{q} - \mathbf{w}|)(\mathbf{q} - \mathbf{w}) = \mathbf{d}/dt \]  
and we have the following stochastic equation for \( \mathbf{v} \):
\[
\frac{d\mathbf{v}}{dt} = \Phi(\mathbf{e}, |\mathbf{q} - \mathbf{w}|)(\mathbf{w} - \mathbf{v}) + \frac{\partial \Phi(\mathbf{e}, |\mathbf{q} - \mathbf{w}|)}{\partial \mathbf{e}} \Delta \mathbf{e}(\mathbf{q} - \mathbf{w}). \]  
(2.9)

The quantities \( \omega \) and \( \Delta \mathbf{e} \) are not independent. Moreover, \( \Delta \mathbf{e} \) is directly related to fluctuations in the number of particles in a given volume, so the mode of fluctuation of \( \Delta \mathbf{e} \) is determined by the statistical properties of the particle system; \( \Delta \mathbf{e} \) is a functional of \( f \), and so \( a_*^* \) and \( B_{a\beta} \) are functionals of \( f \), and (1.6) resembles the kinetic equations of self-consistent fields [8]. Relation (2.9) shows that the statistical characteristics of \( \mathbf{v}(t) \) may have pronounced anisotropy in the general case. For the isotropic state, \( \mathbf{G} = \mathbf{q} = \mathbf{w} = 0 \) and
\[
\frac{d\mathbf{v}}{dt} = - \Phi(\mathbf{e}, 0) \mathbf{v} + \Phi(\mathbf{e}, 0) \mathbf{w}, \quad \mathbf{v} = \mathbf{u}. \]  
(2.10)

Then
\[
a_\alpha = \Phi(\mathbf{e}, 0) u_\alpha, \quad B_{a\beta} = B_0(\mathbf{e}) a_{a\beta}. \]  
(2.11)

If the components are in relative motion, the condition for conservation of the mean flow rate of the fluid gives
\[
\mathbf{v} = \mathbf{v}_1 + V \frac{\mathbf{q} - \mathbf{w}}{|\mathbf{q} - \mathbf{w}|}, \quad \omega = \omega_1 - \frac{\Delta \mathbf{e}}{\mathbf{e}}(\mathbf{q} - \mathbf{w}), \]  
(2.12)
in which \( \omega_1 \) is uniformly distributed with respect to direction and \( |\mathbf{v}_1| \sim |\mathbf{v}| \ll |\mathbf{V}| \).

The stochastic equation for \( \mathbf{v}_1 \) and \( V \) is
\[
\frac{dV}{dt} = - \Phi(\mathbf{e}, |\mathbf{q} - \mathbf{w}|) V. 
\]  
(2.13)