On-Line Estimation of Nonlinear Physical Systems

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Recursive algorithms for estimating states of nonlinear physical systems are presented. Orthogonality properties are rediscovered and the associated polynomials are used to linearize state and observation models of the underlying random processes. This requires some key hypotheses regarding the structure of these processes, which may then take account of a wide range of applications. The latter include streamflow forecasting, flood estimation, environmental protection, earthquake engineering, and mine planning. The proposed estimation algorithm may be compared favorably to Taylor series-type filters, nonlinear filters which approximate the probability density by Edgeworth or Gram–Charlier series, as well as to conventional statistical linearization-type estimators. Moreover, the method has several advantages over nonrecursive estimators like disjunctive kriging. To link theory with practice, some numerical results for a simulated system are presented, in which responses from the proposed and extended Kalman algorithms are compared.

KEY WORDS: Random processes, factorable model, recursive nonlinear estimation.

INTRODUCTION

Many physical processes cannot be represented adequately by traditional linear models. Flood Estimation (Mein et al, 1974), stream flow forecasting (Bras and Georgakakos, 1980), and soil parameters identification (Christakos, 1985) are a few examples involving nonlinear systems. Nonlinearities also arise in mine planning and recoverable reserves estimation (Kim et al, 1977; Jackson and Marechal, 1979).

Given the success in linear real-line problems, certain nonlinear recursive optimization algorithms use related approaches in nonlinear systems with behavior close to that of linear ones. Assuming that nonlinearities can be expanded in Taylor series, the resulting extended Kalman equations for the estimate and error variance may be obtained (Gelb, 1974; Anderson and Moore, 1979). Work by Athans et al, (1968), suggests the expansion of the state nonlinearity about the optimal estimate, but the residual of the expansion is unspecified. Then, by...
establishing a cost criterion, one may derive this residual recursively. Similar analyses have been done by Jaswinski (1966) and Culver (1969). Due to the Taylor-series expansions these methods require that the nonlinear functions of interest are differentiable. Thus, in order to take into account the effect of saturation, threshold, and other nonlinearities, any discontinuities or corners in the nonlinear functions may be approximated, and this reduces the accuracy of resulting expansions.

Some authors (e.g., Sorenson and Stubberud, 1968; Willman, 1981) have viewed the problem of nonlinear estimation as one of approximating the probability density for the state conditioned on all available measurement data by Edgeworth or Gram-Charlier series. This procedure leads to complicated equations describing the moments of the density; analytic criteria for judging the validity of the approximations also are difficult to obtain.

Nonlinear recursive optimization based on statistical linearization (Mahalanobis and Farooq, 1971; Gelb 1974) does not make any assumption regarding function differentiability but, in order to calculate the expansion coefficients, does require knowledge of the full distribution of the state at every step. Furthermore, much calculation is needed depending upon the specific type of the nonlinearities. Therefore, only the first few terms in the expansions usually are kept, and this further questions the accuracy gained.

In an attempt to treat nonlinearities which have arisen in mining geostatistics, Matheron (1976) introduced disjunctive kriging. This is a nonrecursive, nonlinear estimator of the form

$$\hat{x}_t = \sum_{i=1}^{n} f_i(x_i)$$

where the random process $\hat{x}_t = \hat{x}(t)$ is the estimator of $x_t = x(t)$ at point $t$, $x_i = x(t_i)$ are samples available at points $t_i$, $i = 1, \cdots, n$, and $f_i(\cdot)$ are measurable functions. An assumption is made that $x_t$ arises from a process $z_t$ with Gaussian uni- and bivariate distributions via some transformations; no measurement model is present. Unknown functions $f_i(\cdot)$ are determined in terms of Hermitian polynomials. Restrictions associated with nonrecursive estimation are (Rendu, 1980; Christakos, 1985): (1) It assumes a fixed amount of data (if new data become available much calculation must be repeated). (2) Disjunctive kriging cannot be used if stationarity is not satisfied (the mean must be known and constant). (3) It relies solely on covariance-type information and may ignore important structural features of the underlying geoprocesses. (4) Additional cost may result in order to determine the optimal number of Hermite polynomials to be used.

In this presentation, a model-based processor is developed that considers expansions of the state and measurement nonlinearities in terms of orthogonal polynomials. These expansions experience properties of significant importance when the states are modeled as factorable random processes. The latter institute