\[ L = a \exp 3m_0 + ia \exp 3t_0, \]
\[ \omega_0 = c \exp 2m_0 + c \exp 2t_0. \]

b) \[ \Psi = \kappa z^2 + pz, \quad \kappa + p = 1, \quad \kappa p = 0, \]
\[ \Phi = (p \exp \sqrt{b} z + \tau \exp - \sqrt{b} z) \exp i \phi, \]
\[ \theta = 3 \kappa z^2 - b r (\kappa z^2, 3 + p z^2; 2) \div i \phi + \kappa. \]

c) \[ \Psi = p \exp \sqrt{b} z + \tau \exp - \sqrt{b} z, \quad n_0 = \text{const}. \]

a) \[ \Phi = (\tau \exp \sqrt{b} z + \omega \exp - \sqrt{b} z) \exp i \phi, \]
\[ \theta = r z \Psi + \tau \exp \sqrt{b} z + \beta \exp - \sqrt{b} z, \]

b) \[ \Phi = (\alpha z + \tau) \exp i \phi, \]
\[ \theta = r z \Psi + \tau \exp \sqrt{b} z \exp - \sqrt{b} z, \]

d) \[ \Psi = p \exp \sqrt{b} z + \tau \exp - \sqrt{b} z, \]
\[ \Phi = (\kappa \Psi + \tau \exp \sqrt{b} z \exp - \sqrt{b} z), \]
\[ \theta = z \left( (\kappa \Psi + \tau \exp \sqrt{b} z \exp - \sqrt{b} z) + \tau \Psi \right) + \kappa \exp (+ \sqrt{b} z) \exp - \sqrt{b} z. \]

In conclusion we note that all solutions presented above are of the Petrov type N.

LITERATURE CITED

EVOLUTION OF ELECTROMAGNETIC WAVES OF A POINT SOURCE IN THE SCHWARZSCHILD FIELD

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The problem of electromagnetic radiation of a source in the gravity field of a nearby black hole is considered. A complete class of exact axial-symmetric solutions of the eikonal equations is found. The components of the field tensor are obtained in the large frequency approximation. An expression is found for the strength of the observed radiation as a function of the distance between the black hole and the source, and of the angle between the directions black hole–observer and black hole–source. It is shown that the system consisting of a black hole and a normal star in rotation around the common center of gravity must look like an object of variable brightness.

It is known that a strong gravitational field acting in the neighborhood of a black hole, causes deviation of the light rays, which can influence the angular distribution of the intensity of the radiation of a nearby star. It was shown in [1] that on the focal line, the intensity of light goes up due to the focusing properties of the gravitational field. It follows from this result that a system consisting of a black hole and a normal star, in rotation around their common center of masses, must be observed as an object of variable brightness, even if the normal star is calm.

In order to find the numerical expression of this effect, one has to obtain the exact solution of the system of Maxwell equations in the gravitational field. However, solving this system of equations in a Schwarzschild field by means of expanding functions in a series in powers of \( r^{-1} \) is complicated by the fact that the gravitational field near the event horizon surface cannot be considered as weak. The electromagnetic pulses in this area of space suffer considerable perturbations, which are conserved even after the pulses have gone a large distance from the black hole. In spite of this difficulty, a number of interesting results...
were obtained relative to the state of the electromagnetic field near a black hole. In [2, 3] it was shown, for example, that the problem of propagation of the initially flat electromagnetic wave in the Schwarzschild field can be reduced to the problem of solving one second-order differential equation with separating variables. The authors of [4] considered solving the equations obtained by the WKB method in [2], and in the weak field approximation, in [5]. In [6], the Debye potentials were found in the immediate vicinity of the horizon. The existence of several images of an isolated far-away source was discovered in [7, 8]. Unfortunately, the transition from several images observed at $r \approx 2m$ to two images observed when $r \to \infty$, remained unclear. The frequency shift registered by an observer in free fall in the vicinity of a horizon, was studied in [7, 9]. The wave field was found to be a superposition of fields of different frequency. However, no qualitative explanation was given to this interesting result. In [10], a particular solution of the eikonal equation in a Schwarzschild field was found, which allowed them to find the wave transition and reflection coefficients with respect to the surface $r = 3m$.

Since the problem of studying the electromagnetic field in a Schwarzschild field is complicated and cannot be solved exactly in the near future, one has to limit oneself to some frequency approximation. For practical considerations, it is most interesting to find solutions holding in the entire space for large frequencies. These are the solutions we shall be considering below.

Let us start with the Maxwell system of equations in a curved space: $\nabla_\lambda F^{\mu \lambda} = 0$, $\nabla_\mu F^{\nu \lambda} + \nabla_\lambda F^{\nu \mu} + \nabla_\nu F^{\lambda \mu} = 0$. We can eliminate the dependence of $F^{\mu \nu}$ from $t$ and $\varphi$ by the substitutions

$$
F^{\mu \nu} = K^{\mu \nu} e^{\rho \xi(t-\chi)},
K^\mu = \sin \varphi L^\mu, K^{\varphi} = \cos \varphi L^\varphi,
K^{\chi} = \sin \varphi L^\chi, K^{\varphi \chi} = \cos \varphi L^{\varphi \chi}, L^\varphi = L^\varphi(\varphi, \theta).\tag{1}
$$

The eikonal $\psi = \psi(t - \chi)$ satisfies the equation $g^{\mu \nu} \nabla_\mu \psi \nabla_\nu \psi = 0$. In the case of an axial symmetry, we shall choose the system of coordinates in such a way that $\partial_\varphi \psi = 0$. Then the eikonal equation can be satisfied by the substitutions [11]

$$
\partial_\chi \chi = -\frac{\cos \varphi}{1 - \frac{2m}{r}}, \partial_\tau \chi = \frac{r^2 \sin \varphi}{\sqrt{r^2 - 2mr}}.\tag{2}
$$

In order to find the equation for $\tau$, we shall compute the quantities $\partial_1 \partial_2 \chi$ and $\partial_2 \partial_1 \chi$ from Eq. (2) in two different ways, and equate them. We obtain

$$
\sin \frac{\partial_\tau}{\partial \theta} = \cos \tau \sqrt{r^2 - 2mr} \frac{\partial_\tau}{\partial r} + \sin \tau \frac{r - 3m}{\sqrt{r^2 - 2mr}}.\tag{3}
$$

In Eq. (3), the unknown function $\tau$ depends on $r$ and $\theta$. To solve Eq. (3), we shall consider $\theta$ as the unknown function, and $r$ and $\tau$, as its arguments. To do this, we perform the following substitutions in Eq. (3):

$$
\frac{\partial_\tau}{\partial \theta} = \frac{1}{\partial_\theta \partial_\tau}, \frac{\partial_\tau}{\partial r} = -\frac{\partial_\theta \partial r}{\partial_\theta \partial_\tau}.\tag{4}
$$

This results in a first-order linear differential equation:

$$
-\cos \tau \sqrt{r^2 - 2mr} \frac{\partial_\theta}{\partial r} + \frac{r - 2m}{\sqrt{r^2 - 2mr}} \sin \tau \frac{\partial_\theta}{\partial \tau} = \sin \tau.\tag{5}
$$

Let us introduce a new function $\xi$:

$$
\xi = \frac{2r^2 \sin \varphi}{\sqrt{r^2 - 2mr}}.\tag{6}
$$

Now it is easy to see that the general solution of Eq. (5) has the form

$$
\theta = \int_{\hat{\delta}}^{r^{-1}} \frac{dx}{\sqrt{\frac{4}{\xi^2} - x^2 + 2m^2}}, \tag{7}
$$

where $f_1(\xi)$ is an arbitrary function of its argument. The expressions (2), together with (6) and (7), give the general solution of the eikonal equation in a Schwarzschild field. The authors of [10] argued that because there is a strong light scattering near the surface $r = 3m$,