Nonlinear Programming: Global Use of the Lagrangian

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Abstract. In some constrained nonlinear programming problems possessing several local optima, a local optimum can be recognized as the global optimum by looking closely at the Lagrangian, the augmented function. Similarly, classes of constrained optimization problems, such as geometric programming problems, can be recognized as possessing at most a single local optimum.

1. Introduction

The continuous, nonlinear, constrained optimization problem is to find 

\[ x = (x_1, \ldots, x_n) \]

in order to

\[ \begin{align*}
\text{minimize} & \quad F(x), \\
\text{subject to} & \quad G_j(x) \leq b_j, \quad j = 1, \ldots, m.
\end{align*} \]

Here, the functions \( F \) and \( G_j \) are assumed to be twice differentiable and the constants \( b_j \) are real numbers.

Figure 1 illustrates the following two-dimensional problem:

\[ \begin{align*}
\text{minimize} & \quad F(x) = -(x_1 - \frac{1}{2})^2 - (x_2 - \frac{1}{2})^2, \\
\text{subject to} & \quad G_1(x) = x_1^2 + 99x_2^2 - 100 \leq 0, \\
& \quad G_2(x) = 99x_1^2 + x_2^2 - 100 \leq 0.
\end{align*} \]
Notice that point \( x^* \) has the property that it is better than any nearby feasible point. Point \( x^* \) is a local optimum. Point \( x^{**} \) is another local optimum. Moreover, \( F(x^{**}) > F(x^*) \). Point \( x^{**} \) is the global optimum.

Nonlinear programming algorithms, such as those of Refs. 1–4, for solving (1) have the capability of numerically locating a local optimum. However, the recognition of whether or not a local optimum is actually the global optimum cannot be handled numerically in the general case. In the absence of knowledge of special properties of the functions \( F \) and \( G_j \), one can only resort to computing \( F \) and \( G_j \) at points of a fine grid which covers the feasible region. Global optimization can be effectively accomplished by other methods in some special problems which possess many local optima (see, for example, Refs. 5–7).

A well-known result (see, for instance, Ref. 8, p. 93) tells us that a large