Identification of Linear Systems Using Long Periods of Observation

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Abstract. A new method is presented for identifying parameters in a linear differential system arising, e.g., from compartment models in drug kinetics. The linearity of the system is used to produce a series of recurrence relations that help reduce the computational load. The method is especially useful when a long period of observation is used to identify the parameters. Numerical experiments are described.

1. Introduction

In studies of the time history of a drug in the human system, it is frequently adequate to consider linear compartment models (Ref. 1). It is assumed that the initial dose instantaneously raises the concentration of the compartment into which it is injected. Similar behavior is also assumed for every additional dose. Between doses, the process is described by a linear, homogeneous, vector-matrix differential equation with constant coefficients (Ref. 2), that is,

\[ \dot{x} = Ax \] (1)

Some of the elements of the matrix \( A \) depend upon various parameters \( \alpha_1, \ldots, \alpha_m \), whose precise values are not known. Observations are made in the process just before each new dose is administered. It is desired to estimate the...
values of the unknown system parameters on the basis of the dynamical measurements made on the system. In our development, it is assumed that the first component of $x$ (that is, $x^1$) is observed at times $t = 1, 2, ..., N$.

The method presented here is a combination of the Newton–Raphson iteration method plus a series of recurrence relations that help reduce the computational work when the period of observations is very long. Furthermore, the storage requirements of the proposed procedure are small. This is because no storage of previous approximations, as in quasilinearization, is needed. The results of numerical experiments are presented.

2. Formulation of the Problem

Consider a system, the state of which at time $t$ is the vector $x$. The dimension of $x$ is $r$. For all times $t \neq i$, $i = 1, 2, ..., N$, the vector $x$ satisfies the linear system of ordinary differential equations

$$\dot{x} = Ax$$

where $A$ is a constant $r \times r$ matrix. The vector $x$ fulfills a jump condition at $t = i$, that is,

$$x(i + 0) - x(i - 0) = d_i, \quad i = 1, 2, ..., N - 1$$

(3)

This condition simulates the instantaneous injection of a drug dose in the system at equispaced intervals. The initial condition is

$$x(0) = c$$

(4)

The matrix $A$ depends upon an $m$-dimensional vector $\alpha$, that is,

$$\alpha = (\alpha_1, ..., \alpha_m)$$

(5)

The first component of $x$ is assumed to be continuous for all time, that is,

$$d_i^1 = 0, \quad i = 1, 2, ..., N - 1$$

(6)

Noisy observations of this component are made at the times $t = 1, 2, ..., N$, and are denoted $b_1, ..., b_N$, respectively.

The identification problem is to estimate the system parameters $\alpha_1, ..., \alpha_m$ on the basis of these observations when $N \gg m$. In this case, storage of solutions of the system (2) with conditions (3)–(4), for an approximation of $\alpha$,