Integrated System Modeling and Optimization Via Quasilinearization

YACOV Y. HAIMES and DAVID A. WISMER

Communicated by C. T. Leondes

Abstract. System modeling and system optimization are two coupled and strongly related concepts in the modern approach to large-scale systems. Yet, they have been treated as two separate problems in the literature. The identification of system parameters, often referred to as system modeling, is essential in order to obtain an optimal control policy. This work considers the two problems jointly and provides a computational methodology in tackling the integrated problem formulation. This is done by viewing one of the objective functions in the bicriterion problem formulation as a constraint. A computational strategy such as quasilinearization is employed for the solution of the integrated problem. An example problem is introduced, and numerical results using an IBM 360/91 digital computer are presented.

1. Introduction

The problems of system modeling and system optimization have been treated as two separate problems in the literature. In each case, the references are too vast to mention here. However, extensive bibliographies are given in the recent survey articles by Peterka and Balakrishnan (Ref. 1) and Westcott (Ref. 2). System identification, being one aspect of system modeling, has been previously shown to strongly interact with system optimization (Refs. 3-4).

1 Paper received December 23, 1970. The authors are very grateful to Professor C. T. Leondes for his invaluable assistance, guidance, and comments. This research was supported in part by the Air Force Office of Scientific Research, Grant No. 699-67, and in part by the National Science Foundation, Grant No. GK-4086.

2 Associate Professor of Engineering, Case Western Reserve University, Cleveland, Ohio.

3 Manager, Operations Research Division, Systems Control, Palo Alto, California.

Any mathematical model consists of unknown variables and known parameters characterizing the system. In general, these parameters are not known exactly, but rather are estimated or determined under nonoptimal conditions; accordingly, the solution generated from such system models is nonoptimal. Hence, a joint treatment of the two problems is essential for a truly optimal solution to the overall problem. Several approaches for tackling the integrated problems have been proposed in the above-cited work. These are the vector-minimization approach, a two-step approach, the \( \epsilon \)-constraint approach, and the minimax approach.

2. System Identification Optimization Problem

Mathematical models, which aim at representing the real physical systems in quantitative form, have become important tools in the design, synthesis, analysis, operation, and control of complex systems. The nature of the physical system under consideration determines which class of mathematical model will closely represent it.

If the response of both the real system and the mathematical model to the same signal inputs is identical (ideally), then the mathematical model simulating the system is considered to be indeed \textit{perfect}. In general, however, these two responses are not identical, and an error exists. Thus, the purpose in system modeling is to construct a mathematical model such that the above difference error is minimized.

The following problems arise in system identification of dynamic models represented by ordinary differential equations. The form of the differential equations is known; that is, the order and the degree of the differential equations is known. However, the coefficients (parameters) and/or the initial conditions are unknown; or, more generally, the order, the coefficients, and/or the initial conditions of the differential equations are unknown. In this work, ordinary differential equations with unknown coefficients and unknown initial conditions are treated.

Consider the following optimization problem:

\[
\min_{\alpha} \int_{t_0}^{t_f} f(X, U, \alpha, t) \, dt, \quad (1)
\]

subject to the constraints

\[
g(X, U, \alpha) \leq 0, \quad (2)
\]

\[
\dot{X} = F(X, U, \alpha, t), \quad X(t_0) = X_0, \quad (3)
\]