On Optimal Stochastic Midcourse Guidance

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Abstract. Optimal stochastic midcourse guidance correction programs previously considered are open-loop, in the sense that feedback from midcourse observations is not included in the gains matrix. This problem is rigorously analyzed here. It is shown how feedback from observations is introduced into the midcourse correction program and how an improved feedback solution can be obtained as the solution of a deterministic optimal feedback control problem. A simple example is solved in complete detail and the open-loop and optimal feedback controls are compared.

1. Introduction

The problem of optimizing the midcourse guidance correction program in a statistical setting has received considerable attention from a number of investigators (Refs. 1–6). In general, the procedure is to linearize the motion about a preplanned, free-fall trajectory which meets the desired objectives. The Kalman–Bucy filter is assumed for the estimator of the state deviations, and the midcourse correction is taken to be linear in the state deviation estimate. Thus, if \( \delta u_t \) is the correction, \( \delta x_t \) the state deviation, and \( \delta \hat{x}_t \) its estimate,

\[
\delta u_t = -A(t) \delta \hat{x}_t
\]

is the class of control corrections considered. The feedback gain matrix \( A \) is sometimes predetermined up to a scalar multiple (Refs. 2 and 6). In any case, \( A \) is taken to be a fixed (nonrandom) function of time.

The resulting stochastic optimization problem thus features linear system equations, linear noisy observations, and linear control. The perform-

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ance index is nonquadratic, however, since it involves an integral of the absolute value of the control. This corresponds to minimizing the fuel (velocity increment) for the midcourse trajectory. As a result, there is no separation of estimation and control, and the control depends upon the covariance of the errors in estimating the state. This feature of the midcourse guidance problem has been demonstrated by the previous authors. These previous studies assume, however, that the initial state deviation estimate $\delta x_0$ is zero and that the control $A(t)$ is a nonrandom time function. The present paper shows that, in general, $A$ is also a function of the midcourse observations through the state deviation estimate $\delta x$, and the covariance of the errors in this estimate.

It is this stochastic nature of the control $A$ which is exploited in this paper. Careful attention to the properties of mathematical expectation (averaging) enables us to improve the optimal control obtained by previous authors. As additional observations are gathered, the control $A$ can be reoptimized. In this way, the control depends upon more data and is improved. More feedback is allowed. In the sense that the control $A$ in previous studies is independent of the observations, it is open-loop. It is optimal for the average (nominal) situation, but is suboptimal for a specific mission. If an analogy is drawn between the solutions of Refs. 1–6 and an open-loop optimal nominal trajectory in a deterministic situation, then the feedback solution discussed here corresponds to the field of extremals in the deterministic case. The theory of the second variation is then applicable.\(^3\) Of course, in the stochastic problem there is considerable latitude in defining feedback information. The control might further be improved if the controller knew a priori that the control will be reoptimized in the future. We also point out that the feedback control discussed in this paper is inferior to what might be termed full feedback, where $\delta u_t$ in (1) is taken as a general, nonlinear functional on all the past observations.

Previous authors considered two different, but related, problems. In Refs. 1, 3–5, the average total velocity correction is minimized subject to a constraint on average square miss. References 2 and 6 minimize a linear combination of average total velocity correction and average square miss. For concreteness, the ideas of this paper are presented in the context of the latter problem and the notation of Refs. 2, 6 is followed. Our theory is, perhaps, more relevant to the former problem. To simplify equations, we consider the problem in continuous time.

The ideas outlined above are graphically displayed in Section 2, where

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\(^3\) Except in the singular problems of Refs. 1, 3, 4.