Optimal and Suboptimal Capacity Allocation in Communication Networks

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Abstract. Gomory and Hu (Ref. 1) formulated the optimal allocation of capacities to the links of a communication network as a problem in linear programming. The application of this formulation to the solution of problems of realistic size does, however, require an excessive amount of computation. In the present paper, a slightly different formulation is given. The resulting optimality conditions readily lend themselves to the construction of problems with known optimal solutions, thereby providing suitable examples for the assessment of the efficiencies of approximate methods. An approximate method that has been found highly efficient in many cases is illustrated by an example.

1. Introductory Remark

Although the techniques presented in this paper have wider applicability, they will be discussed in terms of their application to the optimal capacity allocation in a communication network.

2. Problem

The following data are given:

(i) the geometrical layout of the proposed network, showing which nodes (communication centers) are to be connected by links (communication channels);
(ii) for each link $\gamma$, the cost $c_{\gamma}$ of installing the unit capacity along this link (the cost of installing an arbitrary capacity $q_{\gamma}$ being $c_{q_{\gamma}}$); and

(iii) for each pair of nodes $i, j$ and for each of two typical periods (for brevity, called day and night), the day traffic $Q_{ij}$ and the night traffic $Q_{uj}$ between these nodes, i.e., the traffic originating at one of the nodes and terminating at the other one.

The following data are wanted:

(iv) for each link $\gamma$, the capacity $q_{\gamma}$ to be allocated to this link so that the resulting network is capable of handling day-time as well as night-time traffic at minimum total cost of capacity installation; and

(v) for each pair of nodes $i, j$ and for day as well as night, a routing of the traffic between these nodes (including its possible distribution over several routes) such that the net with the capacities $q_{\gamma}$ can take care of all the given traffic if this routing is adopted.

3. Remark

In (ii), a linear relation has been stipulated between the capacity $q_{\gamma}$ of the typical link $\gamma$ and the cost of installing this capacity. The treatment presented in this paper can, however, be generalized to the case of a cost that is an arbitrary nondecreasing function of capacity. This generalization will be left to a subsequent paper.

4. Mathematical Formulation of the Problem

For the problem considered here, there is no need to distinguish between communications from $i$ to $j$ and communications from $j$ to $i$. For simplicity, all traffic between a pair of nodes will therefore be regarded as originating at the node of smaller number, and the label $i, j$ of a node pair will imply that $i < j$.

Along a link $\gamma$ with endpoints $r$ and $s$, some traffic (e.g., the traffic $k, l$) may be directed from $r$ to $s$, while other traffic (e.g., the traffic $m, n$) may be directed from $s$ to $r$. For the purpose of the following analysis, it is therefore convenient to regard the link $\gamma$ as consisting of two directed arcs $\gamma_1$ and $\gamma_2$ that join the nodes $r$ and $s$ but have opposite directions. (The convention will be adopted that, for $r < s$, the arc $\gamma_1$ is directed from $r$ to $s$.) This makes