Optimal Switching in Coplanar Orbit Transfer

NGUYEN X. VINH and ROBERT DUDLEY CULP

Communicated by T. N. Edelbaum

Abstract. The set of all controls that satisfy the Weierstrass necessary condition for optimality in the problem of time-open, coplanar orbit transfer via impulses is presented, along with the switching relations that must be satisfied at a corner in an optimal trajectory. This includes detailed data for eccentricities near unity. This study takes advantage of recently discovered closed-form solutions for the switching surfaces of this problem.

1. Introduction

The recent formulation of closed-form expressions for the switching conditions (Refs. 1–2) has rendered complete the solution of time-open, optimal, coplanar orbit transfer under conditions of unlimited thrust. The advances in this problem were made by applying the Weierstrass necessary condition and the Weierstrass–Erdmann corner condition in order to eliminate all but a small portion of the possible controls as nonoptimal. The results of this procedure for orbits with eccentricity less than 0.925 were presented in Ref. 3, along with a discussion of the method.

This approach may be viewed in a number of ways. It is an application of the maximum principle in a step-by-step manner. It may be considered as the enforcement of the triangle inequality on the characteristic velocity metric in the state space of coplanar elliptical orbits. Thus, the metric is completed by the smallest possible convex hull. The developable surface that

1 Paper received July 8, 1970. Portions of this work were supported by NASA Contract No. NASr-54(06) and by NASA Grant No. NGR-06-003-033.

2 Associate Professor, Department of Aerospace Engineering, The University of Michigan, Ann Arbor, Michigan.

3 Assistant Professor, Department of Aerospace Engineering Sciences, University of Colorado, Boulder, Colorado.
forms this hull has generators which are the optimal switches sought. The previously published results (Ref. 3) did not include the region of eccentricities larger than 0.925 because the switches began to include triple-point switches there. The computation by means of tangent planes was much more complicated. Also, the interest in such a large eccentricity was not high at that time.

The complications in the computation for high eccentricity have been eliminated by the appearance of the closed-form solutions to the switching function. Interest in the large-eccentricity results has increased particularly because of the problem of transition through highly eccentric orbits from a hyperbolic arrival velocity. The previous results (Ref. 3) have proven valuable in constructing a near-optimal controller for terminal orbit tailoring starting from hyperbolic arrival for the Viking mission. This required the extension of the complete numerical data to high eccentricities.

2. Orbit Transfer as a Control Problem

In this orbit transfer problem, there are two control variables, one specifying the position of the vehicle on the orbit and the second specifying the direction of the velocity impulse. The position on the orbit is measured by the true anomaly \( \theta \) counted positive in the direction of the velocity in the orbit. The direction of the impulse is given by the angle \( \psi \) measured up from the local horizontal in the same manner as the flight path angle (Fig. 1). The conditions of optimality considered here are local conditions and depend only on the eccentricity and the control variables. The impulse magnitude enters only to determine the elements of the osculating orbit as the impulse is increased from zero to its final magnitude.

Fig. 1. Geometry of a switching.