An Infeasible Method of Large-System Optimization by Direct Coordination of Subsystem Inputs

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Abstract. An infeasible method of large-system optimization is proposed. The dual gap is resolved by use of the generalized Lagrangian as in the previous methods due to Stephanopoulos et al. and Watanabe et al. The values of subsystem inputs are, however, coordinated in the second level, instead of being adjusted in the first level, as in previous methods. As a result, in contrast with previous methods, the subproblems in the first level include a small number of variables to be adjusted; in addition, the generalized Lagrangian is decomposable in a simple manner. Further, the decomposition is not subject to any restriction, which is often encountered in feasible methods.

Key Words. Decomposition techniques, generalized Lagrangian, large-systems optimization, method of multipliers.

1. Introduction

Several methods of large-system optimization using decomposition techniques have been proposed heretofore. Lasdon has attempted to optimize the whole system by seeking the saddle point of the Lagrangian through two-level optimization composed of (i) minimization with respect to the system variables in each subsystem of the first level and (ii) maximization with respect to the Lagrangian multipliers in the second level (Ref. 1). Unfortunately, the saddle point does not always give the true optimum; in such a situation, it is often said that there exists a dual gap. Although the method of multipliers using the generalized Lagrangian in place of the

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ordinary Lagrangian can resolve the dual gap (Ref. 2), the decomposition of the generalized Lagrangian requires some specialized techniques which result in three-level optimization. Additional coordination variables are introduced, and the saddle point of the generalized Lagrangian is sought through the minimization with respect to the system variables in each subsystem of the first level, through the minimization with respect to additional coordination variables in the second level, and through the maximization with respect to the Lagrangian multipliers in the third level. If an output variable of one subsystem is an input variable of another subsystem, an equality must hold between the input and the output of each such pair, at least at the saddle point. As additional coordination variables, the differences between the input variables and the output variables and the mean value of each pair were chosen by Stephanopoulos et al. (Ref. 3) and Watanabe et al. (Ref. 4), respectively. In this paper, it is proposed to reduce the total number of independent variables as well as the scale of each subproblem in the first level by choosing the input variables of each subsystem as coordination variables.

It is possible to choose as coordination variables both the input variables and the output variables of each subsystem satisfying the necessary equality conditions imposed on them; it is also possible to work on two levels without introducing Lagrangian multipliers (Refs. 5 and 6). Such a method is called the feasible method, in contrast with the infeasible method stated above. By choosing a feasible method, the total number of independent variables can be reduced, but the freedom in decomposing the system is also reduced, in general. In the proposed method, the total number of independent variables seems to be minimized without affecting the freedom in decomposing the system.

2. Problem Formulation

Consider a system which is decomposable into $N$ interconnected subsystems. Denote by $x_i$ and $u_i$ the input vector and the decision vector of the $i$th subsystem, respectively. Denote by $y_i$ the vector composed of such output variables of the $i$th subsystem that are supplied to other subsystems as input variables. Then, the output equation of each subsystem and the interconnection equation between subsystems can be written as

$$y_i = f_i(x_i, u_i), \quad i = 1, 2, \ldots, N,$$

and

$$y_i = \sum_{j=1}^{N} C_{ij} x_j, \quad i = 1, 2, \ldots, N,$$