Feedback Control for a Class of Pest Management Problems Modeled by a Noninteractively Distributed System

W. M. Getz

Communicated by G. Leitmann

Abstract. A class of distributed-parameter optimal control problems whose system dynamics are more akin to those of systems of ordinary differential equations is presented together with a set of necessary conditions for optimality. It is shown that certain insect pest management problems fall within the ambit of this theory, which is then used to investigate the properties of the optimal pesticide application schedule and to synthesize a part-feedback control strategy. This latter strategy is presented in a way which makes it applicable to devising strategies for the application of pesticides in agricultural systems that are too complex to be easily modeled, but for which correlations are available between pest levels at certain points during the growing season and yield losses.

Key Words. Pest management, feedback control, optimal application of pesticides.

1. Introduction

The optimal control of systems governed by partial differential equations or integral equations is, in general, a more difficult problem to deal with (see, for example, Refs. 1 and 2) than optimally controlling systems governed by ordinary differential equations (see, for example, Ref. 3). In the former, certain special cases can be easily dealt with, especially linear problems, and some of these cases have been applied in the area of biological resource management (Refs. 4, 5, 6).

In this paper, necessary conditions will be presented for the optimal control of distributed-parameter system whose evolution through time is

1 Assistant Biomathematician, Departments of Entomological Sciences and Plant Pathology, University of California, Berkeley, California.
uncoupled across the parameter space. Thus, although the dynamic
equations form an infinite system, their solution is more characteristic of
a system of ordinary differential equations. Furthermore, although the
general problem considered herein is nonlinear, it contains no state con-
straints or terminal manifold conditions, so that necessary conditions are
easily derivable following elementary variational arguments (see, for
example, Ref. 3).

An insect pest management problem, in which the number of larvae
emerging in a crop system are determined from the number of eggs present
in that system and from a distribution of hatching rates over time, is shown
to fall within the ambit of the general formulation. The necessary conditions
are then applied to investigate the nature of the optimal control strategy
when the problem is linear. This leads to the derivation of a quasi-feedback
control strategy for the application of pesticides. Although this strategy is
derived under a very idealized set of assumptions, its presentation is such
that it could be applied usefully to more general problems. These include
systems for which no dynamic model exists, but only correlations exist
between larval population levels at certain times during the season and
expected yield loss.

2. Problem Statement and Necessary Conditions

Consider the following fixed-final-time optimal control problem:
Minimize
\[ J[u(\cdot)] = \int_{t_0}^{t_f} \left( L(x, u(t), t, y) \, dy \right) dt + \int_{y \in \Gamma} K(x(t_f, y), y) \, dy, \] (1)
over all functions \( u(\cdot) \) that are piecewise continuous on \([t_0, t_f]\) and assume
values in \( U \subset \mathbb{R}^m \), subject to the system equations
\[ \frac{\partial x}{\partial t} = f(x, u(t), t, y), \] (2)
\[ x(t_0, y) = p(y), \] (3)
which hold for each \( y \in \Gamma \).

Here, \( x \in \mathbb{R}^n \) is a vector of functions \( x_i(t, y) \) which, assuming suitable
growth conditions on \( f \) as a function of \( x \), represent the solution of (2) and
(3) on the domain
\[ D = [t_0, t_f] \times \Gamma, \]
where \( \Gamma \) is taken as a compact subset of \( \mathbb{R}^k \). The vector functions \( f \) and \( p \)
and the scalar functions \( L \) and \( K \) are assumed, where appropriate, to be