A Globally Convergent Version of Robinson's Algorithm for General Nonlinear Programming Problems without Using Derivatives

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Abstract. Robinson's quadratically convergent algorithm for general nonlinear programming problems is modified in such a way that, instead of exact derivatives of the objective function and the constraints, approximations of these can be used which are computed by differences of function values. This locally convergent algorithm is then combined with a penalty function method to provide a globally and quadratically convergent algorithm that does not require the calculation of derivatives.

Key Words. Nonlinear minimization, nonlinear constraints, linear constraints, derivative-free minimization, penalty functions.

1. Introduction

One of the best methods to solve general nonlinear programming problems is Robinson's quadratically convergent algorithm (see Ref. 1). This algorithm constructs a sequence of subproblems with linear constraints, for which efficient algorithms are available. Assuming that these subproblems can be solved exactly and that the process is started sufficiently close to a strict second-order Kuhn–Tucker point, Robinson proved $R$-quadratic convergence of the subproblem solutions to that point.

In Ref. 2, a modification of Robinson's algorithm was presented, which uses only linear equality constraints (no inequalities) and requires only approximate solutions of the subproblems. The convergence results are the same as for Robinson's algorithm. In Ref. 3, this modification was combined with a penalty function method to provide a very efficient globally convergent method.

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In this paper, another modification is presented, which does not make use of any explicit derivative of the objective function or the constraints. In fact, in practice, derivatives are often difficult to calculate or are not available at all. Even if analytical functions can be provided for all partial derivatives, their computer implementation often requires a large amount of effort in problem preparation.

These difficulties are avoided by the algorithms of this paper. First, the locally convergent modification of Robinson's algorithm is changed in such a way that, instead of derivatives, approximations of these can be used which are computed by differences of function values. For this algorithm, the same local convergence properties are shown as for Robinson's original algorithm. Then, this algorithm is combined with a penalty function method, such that global and quadratic convergence can be shown. This algorithm is tested on some numerical problems and shows good results.

2. Formulation of the Problem, Definitions, and Notations

Let $f, h_i(x), i = 1, \ldots, m + p,$ be real-valued functions, $\mathbb{R}^n \rightarrow \mathbb{R}.$ If $f$ is differentiable at a point $x$, we denote its gradient at $x$, by $\nabla f(x)$. We define

$$h_i(x)_+ = \begin{cases} h_i(x), & \text{if } h_i(x) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

For $x \in \mathbb{R}^n$, let $\|x\|$ denote the Euclidean norm and $x'$ the transpose of $x$. We consider the problem

$$\min\{f(x) | h_i(x) \leq 0, i = 1, \ldots, m, h_i(x) = 0, i = m + 1, \ldots, m + p\}. \quad (1)$$

A Kuhn–Tucker point of problem (1) is a point $x \in \mathbb{R}^n$ such that there is $u \in \mathbb{R}^{m+p}$, with

$$\nabla f(x) = \sum_{i=1}^{m+p} u_i \nabla h_i(x),$$

$$u_i h_i(x) = 0, \quad i = 1, \ldots, m,$$

$$h_i(x) = 0, \quad i = m + 1, \ldots, m + p,$$

$$h_i(x) \leq 0, \quad i = 1, \ldots, m,$$

$$u_i \leq 0, \quad i = 1, \ldots, m.$$