Optimal Control of Distributed Nuclear Reactors Using Functional Analysis

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Abstract. The minimum norm formalism of functional analysis is applied to the problem of minimizing a quadratic cost functional that penalizes the control effort and the deviations of the neutron flux distribution throughout the reactor core. The conditions for optimality are derived for a general, linearized, reactor model with a finite number of control rods. These conditions take the form of a coupled and finite set of Fredholm's integral equations of the second kind with nondegenerate kernels. An example is presented in which the homogeneous slab reactor model is considered. A contraction mapping algorithm is proposed to compute the optimal control.

Key Words. Optimal control, distributed parameter systems, nuclear reactors, functional analysis.

1. Background

The first application of the abstract formalism of functional analysis to the problem of controlling the neutron flux distribution in nuclear reactor cores was published by Kyong in 1968 (Ref. 1).

Kyong, in his work, treated the terminal state control problem for a reactor core of cylindrical configuration, containing a finite number of control rods, with the neutron kinetics modeled by the one-group neutron diffusion equation.

The unique optimal control for the terminal problem was shown to satisfy a coupled set of Fredholm's integral equations of the second kind with

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degenerate kernels. A rigorous method, based on the characteristic expansion, was proposed for solving the integral equations.

It became clear from Kyong's work that the functional analytic formulation yields necessary and sufficient conditions for optimality in a form which is amenable to the application of a different class of computational techniques, which in some cases may prove to be superior to the more familiar methods associated with the variational formulation of optimal control problems. These techniques approach the control problem through modal expansion methods (Refs. 2, 3, 4) and invoke variational principles which yield necessary conditions for optimality in the form of an infinite system or ordinary differential equations with mixed boundary conditions; in contrast, the functional analytic approach yields necessary and sufficient conditions for optimality in the form of a finite set of integral equations.

In 1973, Iwazumi and Koga (Ref. 5) also discussed the application of the functional analytic formulation to the terminal state control problem, but this time a slab reactor model with distributed control input and the one group–one delayed precursor neutron kinetics were considered.

Given that, in the terminal state problem, the neutron flux deviations from a desired state are only penalized at the end of the time interval being considered, it is possible that large power transients could result from the implementation of the control function which is optimum for the terminal state problem.

In 1977, Nieva et al. (Ref. 6) applied the minimum-norm formalism of functional analysis to the case where the neutron flux distribution is brought from an initial state to a desired equilibrium distribution, while penalizing both the deviations from equilibrium and the control effort along the trajectory. A suboptimal control was derived by expanding the control functions in terms of a finite set of known functions of the time.

In this paper, we consider the applications of the minimum-norm formalism to the problem of minimizing a quadratic cost functional that penalizes the control effort and the deviations of the neutron flux distribution throughout the reactor core volume. First, the necessary and sufficient conditions for optimality are derived for a general linearized reactor model with a finite number of control rods. These conditions are obtained in the form of a coupled and finite set of Fredholm's integral equations of the second kind with nondegenerate kernels. Later, this result is particularized to the case of a slab reactor model and the one neutron-group diffusion equation. Finally, a contraction-mapping algorithm is proposed to solve the resulting integral equations and the corresponding error bounds are estimated.