Efficiency in Integral Facility Design Problems

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Abstract. An example of design might be a warehouse floor (represented by a set S) of area A, with unspecified shape. Given m warehouse users, we suppose that user i has a known disutility function $f_i$, such that $H_i(S)$, the integral of $f_i$ over the set S (for example, total travel distance), defines the disutility of the design S to user i. For the vector $H(S)$ with entries $H_i(S)$, we study the vector minimization problem over the set {$H(S) : S a design$} and call a design efficient if and only if it solves this problem. Assuming a mild regularity condition, we give necessary and sufficient conditions for a design to be efficient, as well as verifiable conditions for the regularity condition to hold. For the case where $f_i$ is the $l_p$-distance from warehouse dock i, with $1 < p < \infty$, a design is efficient if and only if it is essentially the same as a contour set of some Steiner-Weber function $f_S = \sum_{i=1}^{m} \lambda_i f_i$, when the $\lambda_i$ are nonnegative constants, not all zero.

Key Words. Multiobjective optimization, efficiency, facilities design, design problems.

1. Introduction

We consider the problem of characterizing efficient designs. A design may be a planar set of known total area A, but unspecified shape. Any design

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must be contained in some given planar set \( L \). As an example of a design, let \( S \) be a warehouse floor, and let \( L \) be the lot of land in which the design will lie. Assume that the design will have \( m \) users, with user \( i \) having a disutility function \( f_i \), where \( f_i(x) \) is the disutility of the point \( x \) in \( S \) to user \( i \). For the warehouse problem, \( f_i(x) \) might be the distance (perhaps, weighted by the frequency of use) which user \( i \) must travel or pay to have traveled, in order to pick up an item stored at \( x \).

For a given design \( S \), define \( H_i(S) \), for \( 1 \leq i \leq m \), to be the integral of the function \( f_i \) over the set \( S \), so that \( H_i(S) \) represents the total disutility of the design \( S \) to customer \( i \), while

\[
H(S) = (H_1(S), \ldots, H_m(S))^T
\]

represents the disutility vector of the design \( S \) for all users. We call a design \( S^* \) efficient if, whenever any design \( S \) satisfies

\[
H(S) \preceq H(S^*),
\]

then it must be true that

\[
H(S) = H(S^*).
\]

An efficient design thus solves a multiple-objective optimization problem and is Pareto optimal (Ref. 1). An evaluation of a design as given by \( H_i(S) \) may occur when users are concerned about total disutility. In the warehouse context, the disutility of \( S \) to user \( i \), \( H_i(S) \), might be appropriate when the users have to pay for total operating costs and cannot agree upon a single-valued disutility function. The total operating costs for a particular user might be taken as proportional to the total travel distance due to storing this user's items in the warehouse. Alternatively, under an equal-likelihood assumption, that is, a random storage policy in a warehouse (any item is equally likely to be stored at any location within the warehouse), each total operating cost, when divided by the constant \( A \) (the area of the warehouse), becomes an average operating cost. Under these circumstances, the problem under consideration is then one of minimizing the vector of average operating costs incurred by the users due to item movement within the warehouse. A third interpretation of the design problem is as a multiple objective location problem where the design \( S \) is to be located, \( S \) cannot be idealized as a point, and \( H_i(S) \) is an average distance of \( S \) from the existing facility \( i \).

We remark that, in many cases, efficient designs should best be viewed as design guidelines, rather than as final answers. Further, as is typically the case with multiple-objective optimization problems, many different designs can be efficient, and the problem remains of choosing among such designs. Nevertheless, we feel that knowledge of efficient designs should be of value