Optimal Feedback Control Policies for Stochastic, Distributed-Parameter Systems

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Abstract. This paper considers optimal feedback control policies for a class of discrete stochastic distributed-parameter systems. The class under consideration has the property that the random variable in the dynamic systems depends only on the time and possesses the Markovian property with stationary transition probabilities. A necessary condition for optimality of a feedback control policy, which has form similar to the Hamiltonian form in the deterministic case, is derived via a dynamic programming approach.

Key Words. Feedback control, open-loop control, control policy, Hamiltonian function, distributed parameter systems, random variable, Markov chain, dynamic programming.

1. Introduction

The problem of controlling a stochastic, distributed-parameter system has been studied recently by Aidarous, Gevers, and Installe in Ref. 1. In their work, the discrete-time control of a class of linear stochastic, distributed-parameter systems, given by a discrete integral equation with a quadratic criterion, has been considered, and the optimal feedback point-wise control is derived by using a set of orthonormal basis functions to approximate the infinite-dimensional space. In this paper, we consider a general criterion, not necessarily quadratic, and a class of discrete stochastic, distributed-parameter systems, not necessarily linear, in which the random variable depends only on time and forms a Markov chain.

In Ref. 2, a dynamic programming approach to the maximum principle of distributed-parameter systems is given. Now, we use this dynamic programming derivation to derive a necessary condition, which has a form

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similar to the Hamiltonian form, that is satisfied by any optimal feedback control policy (page 210, Ref. 3) for the class of the stochastic, distributed-parameter systems under our consideration.

In Section 2, we describe our discrete stochastic, distributed-parameter system and the criterion to be minimized. In Section 3, we derive a necessary condition. In Section 4, we give a simple numerical example to illustrate the necessary condition, and we compare the optimal feedback control policy obtained by this condition to the optimal open-loop control policy (page 210, Ref. 3).

2. Problem Formulation

Let us consider the dynamic system given by

\[ v((j + 1) \Delta t, i \Delta x) = v(j \Delta t, i \Delta x) + f(j \Delta t, i \Delta x, v(j \Delta t, i \Delta x), [v(j \Delta t, i \Delta x) - v(j \Delta t, (i - 1) \Delta x)]/\Delta x, u(j \Delta t, i \Delta x), \tau(j \Delta t)) \Delta t, \]

(1)

\[ i = 1, \ldots, M \text{ and } j = 0, \ldots, N - 1, \text{ with initial conditions} \]

\[ v(0, i \Delta x) = v_0(0, i \Delta x), \quad \tau(0) = \tau_0, \]

(2)

\[ i = 1, \ldots, M, \text{ and boundary conditions} \]

\[ v(j \Delta t, 0) = v_0(j \Delta t, 0), \]

(3)

\[ j = 0, \ldots, N - 1. \text{ We assume that } g \text{ and } f \text{ are continuously differentiable with respect to their arguments. We call } \Delta x \text{ the duration of a state and } X = M \Delta x \text{ the final state. Similarly, } \Delta t \text{ is the duration of a stage and } T = N \Delta t \text{ is the final time. For convention, we use the notations } v_{ij} \text{ for } v(j \Delta t, i \Delta x) \text{ and } v_{ij} \text{ for } [v(j \Delta t, i \Delta x) - v(j \Delta t, (i - 1) \Delta x)]/\Delta x, \text{ since the last term approaches } \partial v(j \Delta t, i \Delta x)/\partial x \text{ as } \Delta x \text{ approaches 0. Similarly, for the other notations } u \text{ and } \tau. \text{ The variable } \tau \text{ is a random variable which depends only on the time, and the process } \{\tau_j, j = 0, \ldots, N\} \text{ is a finite-state Markov chain with state space } \{1, \ldots, Q\} \text{ and stationary transition probabilities } P_{ik}, \text{ i.e.,} \]

\[ p_{ik} = P\{\tau_{i+1} = k | \tau_i = l\}. \]

We should note that the system (1) is a discrete version of

\[ \partial v/\partial t = f(t, x, v, \partial v/\partial x, u, \tau), \]

with initial conditions \( v(0, x), \tau(0) \) and boundary condition \( v(t, 0). \)

At each stage, say stage \( j \), we observe the state

\[ \psi_j = (v_{1j}, \ldots, v_{Mj}) \]