Nonmonotone Line Search for Minimax Problems

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Abstract. It was recently shown that, in the solution of smooth constrained optimization problems by sequential quadratic programming (SQP), the Maratos effect can be prevented by means of a certain nonmonotone (more precisely, three-step or four-step monotone) line search. Using a well-known transformation, this scheme can be readily extended to the case of minimax problems. It turns out however that, due to the structure of these problems, one can use a simpler scheme. Such a scheme is proposed and analyzed in this paper. Numerical experiments indicate a significant advantage of the proposed line search over the Armijo search.

Key Words. Minimax problems, SQP directions, Maratos effect, superlinear convergence.

1. Introduction

Consider the minimax problem

\[(P) \quad \min f(x), \]

where

\[f(x) = \max_{i=1,\ldots,p} f_i(x), \]

with \(f_i : \mathbb{R}^n \to \mathbb{R}, \quad i = 1, \ldots, p, \) smooth.

Several authors have proposed, among other approaches (e.g., Refs. 1–3), extensions of the popular sequential quadratic programming (SQP)
scheme, originally proposed for the solution of smooth constrained problems, to the minimax framework (e.g., Refs. 4-9). Global convergence is usually insured by means of a line search, forcing a decrease of $f$ at each iteration.

Typically, under mild assumptions, these algorithms exhibit a local superlinear (or two-step superlinear) rate of convergence provided the step-size is not truncated by the line search when a solution is approached. Unfortunately, it is known that in general the full step does not yield a decrease of $f$, and thus the line search may prevent superlinear convergence to take place (Maratos-like effect). As pointed out by Womersley and Fletcher (Ref. 9) and by Conn and Li (Ref. 3), the watchdog technique (Ref. 10) and the bending technique (Refs. 11 and 12), proposed for circumventing the Maratos effect in the context of smooth constrained optimization, can be easily extended to the minimax framework. Both approaches, however, have drawbacks. The watchdog technique may result in repeated backtracking in early iterations, and the bending technique requires an additional evaluation of $f$ at each iteration.

A few years ago, in the context of Newton's method for smooth unconstrained optimization, Grippo, Lampariello, and Lucidi (Ref. 13) proposed a "nonmonotone" line search according to which the objective function is not forced to decrease at every iteration but merely every $M$ iterations, where $M$ is a freely selected positive integer. They showed that, with such a line search, global convergence is still guaranteed, and they pointed out that, as the full Newton step can then be taken earlier, convergence may often be speeded up. Their numerical tests were indeed very promising. Recently, it was shown that making use of a suitable extension of this scheme to smooth constrained optimization, in the framework of SQP with penalty function-based line search, has the additional advantage of automatically allowing a full step to be taken locally, thus avoiding the Maratos effect (Ref. 14).

Many of the schemes that have been proposed for the solution of minimax problems can be viewed as follows. First problem $(P)$ is replaced by the equivalent smooth constrained problem in $(x^0, x^1, \ldots, x^n) \in \mathbb{R}^{n+1}$,

$$(P') \quad \min \ x^0, \quad \text{s.t.} \quad f(x) \leq x^0, \quad i = 1, \ldots, p,$$

and application of a constrained optimization algorithm to this problem is considered. The resulting iteration is then refined to exploit the structure of the problem. In particular, in the case of sequential quadratic programming, refinements include: (i) line search on $f$ rather than on a penalty function; (ii) constraints made tight at the end of each iteration; (iii) estimation of a Hessian of size $n \times n$ instead of $(n+1) \times (n+1)$. The question thus arises