A Quasi-Discrete Newton Algorithm
with a Nonmonotone Stabilization Technique

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Abstract. In this paper, we define an unconstrained optimization algorithm employing only first-order derivatives, in which a nonmonotone stabilization technique is used in conjunction with a quasi-discrete Newton method for the computation of the search direction. Global and superlinear convergence is proved, and numerical results are reported.

Key Words. Nonlinear programming, unconstrained minimization, Newton-type methods, line search techniques.

1. Introduction

It has been shown in Ref. 1 that the use of a nonmonotone stabilization technique is a convenient strategy for globalizing Newton's method in unconstrained minimization. In fact, this technique allows a relaxation of the usual descent requirements on the sequence of function values, in a way that global convergence is preserved, and yet the unit stepsize is accepted in most of iterations. This has proved to be particularly advantageous from a computational point of view, especially in the solution of difficult test problems which are badly scaled or possess severe nonlinearities.

In this paper, we define an unconstrained optimization algorithm employing only first-order derivatives, in which the nonmonotone stabilization technique of Ref. 1 is used in conjunction with a modified version of the quasi-discrete Newton method proposed in Ref. 2. More specifically, the search direction is computed by minimizing the quadratic approximation of the objective function at the current point, by means of a truncated

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quasi-Newton algorithm with finite difference approximations of second-order derivatives. When enough storage is available, as we assume, the quasi-discrete Newton method has the advantage, with respect to truncated Newton schemes based on the conjugate gradient method (see, e.g., Refs. 3 and 4), that the inverse Hessian approximation obtained at the conclusion of the inner loop can be used as the initial approximation for the next inner loop. In this way, considerable savings in the number of gradient evaluations needed for the computation of an approximate Newton direction can be achieved. On the other hand, in comparison with standard nonlinear quasi-Newton algorithms, the quasi-discrete Newton method may have the advantage of avoiding the need for an accurate line search, and this allows the use of a nonmonotone stabilization technique for ensuring global convergence.

The paper is organized as follows. In Section 2, we describe the nonmonotone stabilization technique of Ref. 1. In Section 3, we define the quasi-discrete Newton scheme employed for the computation of the search direction, and we prove that, under suitable assumptions, an ultimate quadratic convergence rate can be established. Finally in Section 4, we report the numerical results obtained for a set of test problems.

2. Nonmonotone Stabilization (NS) Technique

We consider the problem

$$\min_{x \in \mathbb{R}^n} f(x),$$

where $f: \mathbb{R}^n \rightarrow \mathbb{R}$. We assume that, for a given $x_0 \in \mathbb{R}^n$, the level set

$$\Omega_0 = \{x \in \mathbb{R}^n: f(x) \leq f(x_0)\}$$

is compact and that both the gradient $g(x)$ and the Hessian matrix $H(x)$ of $f$ exist and are continuous on $\Omega_0$.

For the solution of problem (1), we consider an algorithm of the form

$$x_{k+1} = x_k + \alpha_k d_k, \quad k = 0, 1, \ldots,$$

where $x_0 \in \mathbb{R}^n$ is a given starting point, $d_k \neq 0$ is a search direction, and $\alpha_k$ is the stepsize. In the sequel, we adopt the notation $f_k = f(x_k)$, $g_k = g(x_k)$, $H_k = H(x_k)$.

In order to ensure global convergence, we make use of a stabilization strategy based on two criteria: a test on the reduction rate of $\|d_k\|$ and a test on the decrease of $f(x_k)$ after a prefixed number of iterations. We consider as standard points those where the test on $\|d_k\|$ is performed and