Primal-Relaxed Dual Global Optimization Approach

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Abstract. A deterministic global optimization approach is proposed for nonconvex constrained nonlinear programming problems. Partitioning of the variables, along with the introduction of transformation variables, if necessary, converts the original problem into primal and relaxed dual subproblems that provide valid upper and lower bounds respectively on the global optimum. Theoretical properties are presented which allow for a rigorous solution of the relaxed dual problem. Proofs of $\epsilon$-finite convergence and $\epsilon$-global optimality are provided. The approach is shown to be particularly suited to (a) quadratic programming problems, (b) quadratically constrained problems, and (c) unconstrained and constrained optimization of polynomial and rational polynomial functions. The theoretical approach is illustrated through a few example problems. Finally, some further developments in the approach are briefly discussed.

Key Words. Global optimization, quadratic programming, polynomial functions, $\epsilon$-optimal solutions.

1. Introduction

Global optimization of nonconvex programming problems has generated a lot of interest in recent years. Surveys, books, and applications for global optimization are available by Dixon and Szego (Refs. 1 and 2), Archetti and Schoen (Ref. 3), Pardalos and Rosen (Refs. 4 and 5), Torn

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The deterministic approaches for global optimization can be largely classified as: (a) Lipschitzian methods, e.g., Ref. 12; (b) branch-and-bound methods, e.g., Refs. 13-15; (c) cutting plane methods, e.g., Ref. 16; (d) difference of convex (DC) and reverse convex function methods, e.g., Refs. 17 and 18; (e) outer approximation methods, e.g., Refs. 19 and 20; (f) primal-dual methods, e.g., Refs. 21-23; (g) linearization methods, e.g., Ref. 24; and (h) interval methods, e.g., Ref. 25. Recent developments in global optimization approaches can be found in Ref. 11.

In this paper, a primal-relaxed dual approach for global optimization is proposed; earlier versions of this work have appeared in Floudas and Visweswaran (Ref. 26) and Visweswaran and Floudas (Ref. 27). It is related to the work of Geoffrion (Ref. 28) and Wolsey (Ref. 29). It does not require Property (P) stated in Ref. 28, and it differs from the resource decomposition algorithm of Wolsey (Ref. 29) in the way the relaxed dual problem is solved. A statement of the global optimization problem is given in Section 2, while Section 3 presents the relevant part of duality theory; extensive discussion of duality theory for decomposition can be found in Flippo (Ref. 30). Section 4 contains the new theoretical results. Section 5 illustrates the branch-and-bound nature of the proposed algorithm and discusses some properties of the branching that can be used to improve the efficiency of the algorithm. Section 6 describes the global optimization algorithm. Section 7 provides the proofs of finite $\epsilon$-convergence and $\epsilon$-global optimality. The application of the algorithm to two illustrating examples is considered in Sections 8 and 9, while Section 10 contains a geometrical interpretation of the algorithm. Sections 11 and 12 discuss the extensions of Section 4 to quadratically constrained problems and problems with polynomial functions.

2. Statement of the Problem

The global optimization problem addressed in this paper is stated as follows. Determine a globally $\epsilon$-optimal solution of the following problem:

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\begin{align*}
\min_{x,y} & \quad f(x, y), \\
\text{s.t.} & \quad g(x, y) \leq 0, \\
& \quad h(x, y) = 0,
\end{align*}
\]