On the Optimal Control of Minimum Film Thickness

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Abstract. The present paper is concerned with the study of controlling the motion of a bearing so that the thin film of lubricant separating it from its container will have the largest possible thickness. Explicit equations are derived, and the control problem is solved explicitly in some simple cases.

Key Words. Optimal control, existence, numerical approximation, piston rings, journal bearings, hydrodynamic lubrication theory.

1. Introduction

Hydrodynamic lubrication theory is concerned with the separation of two surfaces in relative motion by a thin layer of viscous fluid. These surfaces are kept apart by the pressure generated in the fluid film due to viscous forces. A detailed description of this may be found in Sabiel and Macken (Ref. 1). The shape, extent, and thickness of the fluid film determines, to a large extent, the magnitude of the resultant of this pressure and the load capacity of the bearing.

Much work has been done by McAllister and Rohde (Refs. 2 and 3) in the optimization of the profile of dynamically loaded journal and slider bearings having the greatest load capacity for a given minimum film thickness. Although the existence of mathematically optimum shapes have been shown, they may lead to unsatisfactory designs from a stability point of view as noted in Ref. 2. Thus, the following problem remains: Can one determine a shape function such that the minimum film gap is maximized and provides a satisfactory design.

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In this paper, we consider a different, but related problem, i.e., the problem of determining the periodic bearing surface motion (with zero mean) which maximizes the minimum film thickness of a hydrodynamically lubricated slider bearing. This problem is motivated by research into novel engine configurations which yield optimum engine efficiency (Refs. 3-6). To define such engines requires tradeoff studies between the combustion process, cylinder heat transfer, and mechanical friction. These are all functions of piston speed. The present research begins to address the mechanical friction problem. The bearing described in this paper can be viewed as representing piston rings/skirts in this context; and the maximum minimum thickness problem is closely tied to the minimum power loss problem as seen in Refs. 7-9.

We assume that the moving surfaces are impermeable and that the lubricant separating them is of constant density and viscosity. If \( H(x) \) is the profile of the bearing, that machined on the part, then the pressure distribution \( p_j(x) \) at time \( t \) is that element of

\[
Q = \{ p(x): p(x) \in H_1(-1, 1), p(x) \equiv 0 \text{ over } (-1, 1) \},
\]

which minimizes the integral \( V(p) \) over \( Q \), where

\[
2V(p) = \int_{-1}^{1} \{(H(x)+J(t))^3(p'(x))^2-2p(x)(U(t)H'(x)+J(t))\} \, dx;
\]

here, \( H_1(-1, 1) \) is defined at the end of this section, \( J(t) \) is the minimum film thickness at time \( t \), \( J(t) \) is the rate of change in the minimum film thickness, \( U(t) \) is the velocity of the moving part, zero is taken to be the cavitation pressure, and \( (-1, 1) \) is the projection of the bearing onto the stationary surface, which is assumed to be flat. That \( J(t) \) is the minimum film thickness is expressed as follows:

\[
\min\{H(x)+J(t): x \in (-1, 1)\} = J(t).
\]

We will require the bearing to support a given load \( F(t) \); i.e., the integral of the pressure as \( x \) goes over the interval \((-1, 1)\), \( t \) fixed, is equal to \( F(t) \). Note that \( p_j(x) \) depends on \( t \) through \( U(t), J(t) \), and \( \dot{J}(t) \). More details on this derivation may be found in Ref. 10.

In summary, the function \( p_j(x) \) is that element of \( Q \) for which

\[
V(p_j) = \min\{V(p): p \in Q\}, \tag{1}
\]

with \( J(t) \in (0, \infty), |\dot{J}(t)| < \infty \), and for which

\[
\int_{-1}^{1} p_j(x) \, dx = F(t) \tag{2}
\]

for each \( t > 0 \).