An Optimal Sequence in Multicharacteristics Inspection

S. O. DUFFUAA\textsuperscript{2} AND A. RAOUF\textsuperscript{3}

Communicated by M. D. Intriligator

Abstract. Given a component with $N$ critical characteristics, each characteristic $i$ has a cost of inspection $C_i > 0$ and a probability of rejection $R_i$, $0 < R_i < 1$. An optimal sequence for ordering these critical characteristics for inspection is found. This sequence minimizes the total expected cost of inspection. The implications and the applications of this sequence are examined.

Key Words. Multicharacteristics inspection, optimal sequence of inspection, mathematical induction.

1. Introduction

Multicharacteristics components supplied by vendors have to be inspected. These characteristics are unequal in their effect on the fitness for use. Some of the characteristics are of critical importance and can endanger human life. Such characteristics are generally classified as critical (Ref. 1). The cost of inspecting the characteristics depends upon the type of tests and equipment needed in carrying out quality assurance procedures. The probability of finding each of these characteristics below acceptable levels varies as well. In general, conformance of critical characteristics to the specified standards is evaluated before other characteristics are inspected. To minimize the probability of false rejection (Type 1 error) and false acceptance (Type 2 error), use of repeat inspections has been suggested.

\textsuperscript{1} The authors would like to acknowledge the assistance provided by the Department of Systems Engineering, King Fahd University of Petroleum and Minerals, in the preparation of this manuscript.

\textsuperscript{2} Associate Professor, Department of Systems Engineering, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia.

\textsuperscript{3} Professor, Department of Systems Engineering, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia.
(Ref. 2), i.e., the use of inspection redundancy. More than one inspection is likely to reduce the expected cost due to Type 1 and Type 2 errors, but will increase the cost of inspection. In Ref. 3, a model is developed which minimizes the total expected cost due to Type 1 errors, Type 2 errors, and repeat inspections. In cases of this type, the inspection of components is discontinued when one of the critical characteristics is found to be defective at any stage in the cycle of inspection. Such components are then returned to the vendors.

The bulk of the total cost of inspection of a component usually is made up of the costs of inspecting critical characteristics. Given a component with several critical characteristics, each characteristic $i$ has a cost of inspection $C_i > 0$ and a probability of rejection $R_i$, with $0 < R_i < 1$; by this, we mean that characteristic $i$ has a probability $R_i$ of being defective. The problem is to find the optimal sequence for inspecting the critical characteristics in order to minimize the total expected cost of inspection. In Ref. 3, a rule has been suggested for generating such a sequence. No proof of this rule was offered by the authors. In this paper, we will prove, in general, that the suggested rule generates the optimal sequence which minimizes the total expected cost of inspecting a component with critical characteristics.

In Section 2, we will define the problem precisely and in more general terms. In Section 3, we will state and prove the main theorem of this paper; in Section 4, we will discuss the implications of this theorem and conclude this paper.

2. Statement of the Problem

We are given a component with $N$ critical characteristics such that each characteristic has to be inspected separately and assumed to be independent of each other. The order in which these characteristics are inspected affects the total cost of inspection. An optimal sequence of inspection will minimize the cost of inspection.

In general, the problem that we are concerned with is the inspection of these critical characteristics. Each critical characteristic $i$ has a cost of inspection $C_i > 0$ and probability of rejection $R_i$, with $0 < R_i < 1$. We intend to study the optimal way of sequencing these characteristics for inspection in order to minimize the total expected cost for inspecting a component, given that a component will be rejected if any characteristic is found defective. Also, it is assumed that the rejection of a component is independent of the order in which its characteristics are inspected.

For small values of $N$, $N \leq 4$, it is feasible to compute the total expected cost for all $N!$ sequences, and from that we can find the optimal ordering.