TECHNICAL NOTE

Existence of Solutions for Integral Inclusions of Urysohn Type with Nonconvex-Valued Orientor Field

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Abstract. In this paper, we prove the existence of solutions for an integral inclusion of Urysohn type with nonconvex orientor field and with delay. We make standard boundedness and continuity assumptions on the data, and we assume that the orientor field is l.s.c. in the state variable. Using a selection theorem of Fryszkowski, we are able to prove the existence of solutions, extending an earlier result of Angell.

Key Words. Lower semicontinuous multifunctions, Aumann's selection theorem, Urysohn equation, Schauder fixed-point theorem, Arzela-Ascoli theorem.

1. Introduction

In a recent paper, Angell (Ref. 1) studied integral inclusions of the Urysohn type with a convex-valued orientor field. The purpose of this note is to extend Angell's work to the case where the orientor field is nonconvex valued.

Integral inclusions appear in the study of control systems, when the deparametrization technique is applied. Working with the integral inclusion is easier to establish the compactness of the set of admissible states, which is crucial in the solution of various optimization problems.

Also in two papers, Glashoff and Sprekels (Refs. 2, 3), illustrated how set-valued Hammerstein inclusions appear naturally in the study of the thermostatic regulation problem, in which the heating devices controlling the temperature of the system are governed by a relay switch.

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Finally, integral inclusions arise in the study of nonsmooth dynamical systems, of reaction-diffusion systems, and of closed-loop systems arising in the synthesis of the optimal control.

2. Preliminaries

Let $(\Omega, \Sigma)$ be a measurable space, and let $X$ be a separable Banach space. Throughout this note, we will use the following notation:

$$P_f(X) = \{A \subseteq X: \text{nonempty, closed}\}.$$

A multifunction $F: \Omega \rightarrow P_f(X)$ is said to be measurable if, for all $x \in X$, $\omega \rightarrow d(x, F(\omega)) = \inf\{\|x - z\|: z \in F(\omega)\}$ is measurable. Let $\mu(\cdot)$ be a finite measure on $(\Omega, \Sigma)$. We define $S^I_F$ to be the set of integrable selectors of $F(\cdot)$, i.e.,

$$S^I_F = \{f(\cdot) \in L^1(X): f(\omega) \in F(\omega), \mu\text{-a.e.}\}.$$

Using this set, we can define a set-valued integral for $F(\cdot)$ by setting

$$\int F = \left\{\int f(\omega) \, d\mu(\omega): f \in S^I_F\right\}.$$

We say that $F(\cdot)$ is integrably bounded if and only if $F(\cdot)$ is measurable and $\omega \rightarrow |F(\omega)| = \sup\{\|z\|: z \in F(\omega)\} \in L^1$. It is easy to see that, in this case, $S^I_F \neq \emptyset$ and so

$$\int F(\omega) \, d\mu(\omega) \neq \emptyset.$$

Let $Y, Z$ be Hausdorff topological spaces. A multifunction $G: Y \rightarrow 2^Z \setminus \emptyset$ is said to be lower semicontinuous (l.s.c.) if, for all $U \subseteq Z$ open, the set

$$G^-(U) = \{y \in Y: G(y) \cap U \neq \emptyset\}$$

is open in $Y$. When $Y$ and $Z$ are first countable, then this is equivalent to saying that, for every $y_n \rightarrow y$ in $Y$, we have

$$G(y) \subseteq \lim\inf G(y_n) = \{z \in Z: z = \lim z_n, z_n \in G(y_n), n \geq 1\}.$$

Recall that, if $Z$ is a metric space with metric $d(\cdot, \cdot)$, then

$$\lim_n G(y_n) = \{z \in Z: \lim_{n \rightarrow \infty} d(z, G(y_n)) = 0\}.$$