TECHNICAL NOTE

Useful Monte Carlo Optimization

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Abstract. This contribution to the debate on Monte Carlo optimization methods shows that there exist techniques that may be useful in many technical applications.

Key Words. Mathematical programming, Monte Carlo optimization, stochastic methods, Gaussian adaptation.

1. Introduction

In a technical note, Dickman and Gilman (Ref. 1) discuss Monte Carlo optimization as an alternative to deterministic optimization for the solution of mathematical problems. A method due to Conley was used in the investigation. The goal of the note is to “put the Monte Carlo optimization techniques in proper perspective.”

The opinions expressed by the authors of Ref. 1 may be misleading for at least two reasons: (i) the Monte Carlo technique advised by Conley is not very efficient; and (ii) the examples used are not very difficult. Most of the functions and restrictions are linear or almost linear; the highest degree of nonlinearity is 3. On such simple examples, it is almost self-evident that deterministic methods are better than Monte Carlo methods. On the other hand, if the complexity of the problem exceeds some limit, we should expect any deterministic method to become a random method, even though the problem functions are still differentiable. Thus, in order to put things in proper perspective, (i) more efficient Monte Carlo methods should be used and (ii) considerably more difficult test problems must be solved. Unfortunately, more difficult test problems were not available.

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2. Gaussian Adaptation

One efficient method is known as Gaussian adaptation (GA, Refs. 2-4). GA has many areas of application, for instance: minimization of functions that are differentiable, or nondifferentiable, or noisy; maximization of production yield; and discrete optimization.

In short, GA is a stochastic adaptive process that adapts a Gaussian distribution to a region or set of feasible points in parameter space. As a result of the adaptation, GA becomes a maximum dispersion process extending the sampling over the largest possible volume in parameter space while keeping the probability $P$ of finding feasible points at some suitable level. For such a process, it is also possible to show that the function $-P \log(P)$ is a good approximation of the efficiency of the process, which is maximized by $P = \exp(-1)$.

In many applications of GA, it is of interest to compare the feasible points with respect to their production yield imposing statistical variability on the parameters. So, even if GA can be used for minimization of functions, its main interest is to look for solutions that have a central position in a feasible region. In this sense, the problems of Ref. 1 are rather difficult for the GA algorithm, because the solutions having the best criterion values are probably peripheral points, not always suitable for manufacturing purposes.

When using GA for minimization of criterion functions, the feasible region is determined by some level of the criterion which is gradually lowered during the minimization. In this sense, GA is similar to simulated annealing (SA, Ref. 5). The main differences are: (i) a different criterion for accepting a feasible point is used; and (ii) there is no Gaussian adaptation in the SA algorithm. Nevertheless, both methods will accept points on higher level than the last feasible point and both methods gradually decrease the probability of finding high-level points.

3. Test Results

In this note, GA has been used on Test Problems 8.2 and 8.5 of Ref. 1, because these problems were causing difficulties for the Conley algorithms.

On Problem 8.2, the solution $(193.1695, 179.8783, 185.1751, 168.4578)$ was found using 5000 calls to the criterion function. The corresponding criterion function value is $-726.6808$.

On Problem 8.5, the following feasible solutions were found using 10,000 calls for the criterion function and a technique similar to that given in Ref. 4. Even though the global solution was not found, the results are