Self-Bounded Controlled Invariants versus Stabilizability

G. Basile¹ and G. Marro²

Communicated by G. Leitmann

Abstract. Self-bounded controlled and self-hidden conditioned invariant subspaces, recently introduced by the authors for a more direct and neat handling of some fundamental concepts of the geometric approach, such as controllability subspaces, are proved in this paper to be very useful tools also in dealing with synthesis problems with stability requirements.

Definitions concerning stability of invariants and stabilizability of controlled invariants, simple and self-bounded, are first presented and discussed. In particular, it is shown that a more straightforward definition for controlled invariant stabilizability allows a simpler development of the theory. Then, some fundamental results relating self-boundedness to stabilizability are derived. For the sake of completeness, all statements are dualized to conditioned invariants, simple and self-hidden.

Key Words. Controlled invariants, (A, B)-invariants, conditioned invariants, (C, A)-invariants, stability, stabilizability, geometric approach, linear system theory.

1. Introduction

The aim of this paper is to present a review of definitions, algorithms, and properties which allow one to include the requirement of stabilizability in the geometric approach, in order to make their use in new developments of the theory more straightforward and easily understood, and to point out the particular stabilizability properties of the classes of self-bounded controlled invariants and self-hidden conditioned invariants.

¹ Professor, Department of Electrical Engineering, University of Florida, Gainesville, Florida.
² Professor, Department of Electronics, Computers, and Systems, University of Bologna, Bologna, Italy.
In Ref. 1, self-boundedness was introduced and used for a more direct derivation of controllability subspaces, while in Ref. 2 the duality of self-boundedness and self-hiddenness was investigated and its importance in control and observation problems was discussed and emphasized through several examples.

Sections 2 and 3 contain definitions and properties concerning stability of $A$-invariants and stabilizability of $(A, \mathcal{B})$-controlled invariants and $(A, \mathcal{C})$-conditioned invariants. Although stabilizability of controlled invariants has been previously defined by many authors (Refs. 3-5), the definition reported herein is different, since it is based on properties of the system state trajectories under a general control action instead of properties of the system matrices under a state-to-input linear feedback connection. Such a definition is fully consistent with the definition of $(A, \mathcal{B})$-controlled invariants as loci of admissible state trajectories under proper control actions, given by the authors in Ref. 6, which is more general of others referring to the possibility of achieving invariance via a state-to-input feedback.

In Section 4, special stabilizability properties of self-bounded controlled and self-hidden conditioned invariants are pointed out and an application to asymptotic state observers is presented.

Finally, in Section 5 some theorems are proved, in which the lattice-type structure of the set of all self-bounded $(A, \mathcal{B})$-controlled invariants is used to derive necessary conditions for the disturbance decoupling problem by state-to-input feedback with stability requirement and its dual problem.

We conclude with some remarks on notation. Script capital letters, such as $\mathcal{A}, \mathcal{B}, \mathcal{C}$ will denote finite-dimensional vector spaces over the field of real numbers; in particular $\mathcal{A}$ stands for $R^n$, the vector space of $n$-tuples of real numbers. Let $A: \mathcal{X} \to \mathcal{X}$ be a linear map, $\mathcal{B}$ and $\mathcal{C}$ any couple of subspaces of $\mathcal{X}$: the symbol $\mathcal{I}$ will be used for $A$-invariants ($A\mathcal{I} \subseteq \mathcal{I}$), $\mathcal{V}$ for $(A, \mathcal{B})$-controlled invariants ($A\mathcal{V} \subseteq \mathcal{V} + \mathcal{B}$), $\mathcal{F}$ for $(A, \mathcal{C})$-conditioned invariants ($A(\mathcal{F} \cap \mathcal{C}) \subseteq \mathcal{F}$).

Since $\mathcal{I}, \mathcal{V}, \mathcal{F}$ are mostly members of lattices or semilattices, the following modular notation will be used for their universal bounds:

$MN. \mathcal{I}(A, \mathcal{B}) =$ the minimum $A$-invariant containing $\mathcal{B}$;

$MX. \mathcal{I}(A, \mathcal{C}) =$ the maximum $A$-invariant contained in $\mathcal{C}$;

$MN. \mathcal{I}(A, \mathcal{C}, \mathcal{B}) =$ the minimum $(A, \mathcal{C})$-conditioned invariant containing $\mathcal{B}$;

$MX. \mathcal{V}(A, \mathcal{B}, \mathcal{C}) =$ the maximum $(A, \mathcal{B})$-controlled invariant contained in $\mathcal{C}$.

In the following development of the theory, we will take further advantage of modularity of notation in order to specify stabilizability features, so that we will use the additional notation: