Suboptimality and Stability of Linear Distributed-Parameter Systems with Finite-Dimensional Controllers

M. J. BALAS

Communicated by G. Leitmann

Abstract. In order to implement feedback control for practical distributed-parameter systems (DPS), the resulting controllers must be finite-dimensional. The most natural approach to obtain such controllers is to make a finite-dimensional approximation, i.e., a reduced-order model, of the DPS and design the controller from this. In past work using perturbation theory, we have analyzed the stability of controllers synthesized this way, but used in the actual DPS; however, such techniques do not yield suboptimal performance results easily. In this paper, we present a modification of the above controller which allows us to more properly imbed the controller as part of the DPS. Using these modified controllers, we are able to show a bound on the suboptimality for an optimal quadratic DPS regulator implemented with a finite-dimensional control, as well as stability bounds. The suboptimality result may be regarded as the distributed-parameter version of the 1968 results of Bongiorno and Youla.

Key Words. Control theory, distributed-parameter systems, partial differential equations.

1. Introduction

Many current engineering systems display a distributed-parameter nature, i.e., their dynamical behavior is best modeled by partial differential equations. In the development of feedback control theory for such applications, it is essential that the infinite-dimensional distributed-parameter

---

1 This research was supported by the National Science Foundation under Grant No. ECS-80-16173 and by the Air Force Office of Scientific Research under Grant No. AFOSR-83-0124. The author would like to thank the reviewer for many helpful suggestions.

2 Associate Professor, Electrical, Computer, and Systems Engineering Department, Rensselaer Polytechnic Institute, Troy, New York.
system (DPS) be controlled by a finite-dimensional system whose order is determined by the practical constraints of on-line computer implementation.

In the past, our work has expanded on this theme using singular and regular perturbation techniques to determine the closed-loop stability of finite-dimensional controllers with infinite-dimensional DPS; this work is summarized in Refs. 1-2. However, with such techniques, we have not been able to obtain performance bounds in addition to the stability bounds. Yet these are certainly needed by the control system designer to assess the trade-offs inherent in the synthesis of DPS control schemes. In this paper, we will propose a new way to view the finite-dimensional control of linear DPS. We will call this the imbedding method for reasons to be seen later.

In addition to new stability bounds, this approach yields bounds on the suboptimality of quadratic performance indices for DPS. Of course, one never gets something for nothing, and here the quid pro quo is that a deeper knowledge of the residual (unmodeled) subsystem is required to modify the usual finite-dimensional controller in order to obtain the desired performance bounds. Our suboptimality results may be considered the distributed-parameter version of those obtained for lumped parameter systems in Refs. 3-4.

We consider linear DPS described by the following equations:

\[ \frac{\partial v(t)}{\partial t} = A v(t) + B f(t), \quad v(0) = v_0, \]
\[ y(t) = C v(t), \]

where the system state \( v(t) \) is in an infinite-dimensional real Hilbert Space \( H \) with inner product \( (\cdot, \cdot) \) and associated norm \( \|\cdot\| \). The operator \( A \) is an unbounded, linear, time-invariant, differential operator with domain \( D(A) \), a subspace which is dense in \( H \) and contains all sufficiently smooth states which satisfy the boundary conditions of the DPS. The operator \( A \) generates a \( C_0 \)-semigroup \( U(t) \) on \( H \); for more about semigroups, see Ref. 5.

The control is introduced via \( M \) inputs,

\[ B f(t) = \sum_{i=1}^{M} b_i f_i(t), \]

where the actuator influence functions \( b_i \) are in \( H \). The system is observed via \( P \) sensors whose outputs \( y_j(t) \) form the vector

\[ y(t) = [y_1(t), \ldots, y_p(t)]^T \]

and are given by

\[ y_j(t) = (c_j, v(t)), \]