Optimal Control of Delay Systems via Block Pulse Functions

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Abstract. The concept of coefficient shift matrix is introduced to represent delay variables in block pulse series. The optimal control of a linear delay system with quadratic performance index is then studied via block pulse functions, which convert the problems into the minimization of a quadratic form with linear algebraic equation constraints. The solution of the two-point boundary-value problem with both delay and advanced arguments is circumvented. The control variable obtained is piecewise constant.

Key Words. Delay systems, optimization, block pulse functions, linear systems.

1. Introduction

The application of Walsh functions in the optimal control of time-invariant linear systems with quadratic performance index was studied by Chen and Hsiao (Refs. 1–2), who obtained piecewise constant gains for easy implementation. The extension to time-varying systems was given by Chen and Shih (Ref. 3). However, the numerical computation is simplified using block pulse functions instead of Walsh functions with identical results (Refs. 4–5).

In the study of delay systems, Chen and Shih (Ref. 6) introduced shift Walsh matrix for the analysis of linear delay systems via Walsh functions. The method was extended to the parameter estimation of delay systems by block pulse functions (Ref. 7).

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In this paper, an alternative approach is used to study delay systems via block pulse functions. Coefficient shift matrices are used to represent delay variables to convert linear delay differential equations into linear algebraic equations. The quadratic performance index is also approximated by using block pulse functions in a quadratic form in terms of the block pulse coefficients of the state and control variables. Thus, the optimal control problem becomes the minimization of a quadratic form with linear algebraic equation constraint. The computation becomes very simple. It involves only the solution of a set of linear algebraic equations. It is known that optimal control of delay systems leads to the solution of two-point boundary-value problems with both delay and advanced arguments. The block pulse function method circumvents this computational difficulty.

2. Coefficient Shift Matrices

The orthogonal block pulse functions \( \psi_i(t) \) are defined as

\[
\psi_i(t) = \begin{cases} 
1, & \text{if } ih \leq t < (i+1)h, \\
0, & \text{otherwise.}
\end{cases}
\]

A vector function \( x(t) \) of \( n \) components which are square integrable in \( 0 \leq t < mh \) can be represented approximately by a finite block pulse series

\[
x(t) = \sum_{i=0}^{m-1} x_i \psi_i(t) = [x_0, x_1, \ldots, x_{m-1}] \psi(t),
\]

where \( \psi(t) \) is the block pulse vector,

\[
\psi^T(t) = [\psi_0(t), \psi_1(t), \ldots, \psi_{m-1}(t)].
\]

Here, \( T \) denotes transpose. The \( x_i \)'s are the block pulse coefficients of \( x(t) \), as obtained from the orthogonality of the block pulse functions

\[
x_i = \left( \frac{1}{h} \right) \int_{ih}^{(i+1)h} x(t) \, dt.
\]

For a product \( A(t)x(t) \), it is known that (Ref. 9)

\[
A(t)x(t) = [A_0x_0, A_1x_1, \ldots, A_{m-1}x_{m-1}] \psi(t),
\]

where \( A(t) \) is an \( n \times n \) matrix and the \( A_i \)'s are the block pulse coefficients of \( A(t) \).

The integration of the block pulse vector \( \psi(t) \) gives (Ref. 10)

\[
\int_0^t \psi(\tau) \, d\tau \approx H\psi(t),
\]