On the Asymptotic Amplitude of Vibrating Plates

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Summary - The asymptotic behaviour of the amplitude of a vibrating plate is investigated by treating the relevant differential equation as a second-order evolution equation in a Hilbert space.

1. Some preliminaries

Let $D$ be a bounded open set in a two-dimensional euclidean space $\mathbb{R}^2$; $\partial D$ be the boundary of $D$. We assume that $\partial D$ is a smooth curve. Let $J$ denote either of the intervals $[0, \infty)$ or $(-\infty, \infty)$. $H$ be the complex Hilbert space $L_2(D)$ whose inner product and norm are respectively denoted by $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$. Let $u(t)$ be a mapping from $J$ into $H$. In this investigation we are concerned with the asymptotic behaviour of $u(t)$ as $t \to + \infty$ where

$$\frac{d^2 u}{dt^2} + Au = f e^{i \omega t} \quad t \in J, \quad i = \sqrt{-1}$$

(1.1)

$$u|_{t=0} = u_0, \quad \frac{du}{dt} \bigg|_{t=0} = v_0,$$

(1.2)

where $\omega$ is a real constant, $A$ is a partial differential operator generated by

$$Au = \left( \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \right) u \quad (x, y) \in \mathbb{R}^2$$

(1.3)

on a suitable domain in $H$ and $u_0, v_0, f \in H, f$ being independent of $t$. The domain and range of an operator $A$ will be denoted by $D(A)$ and $R(A)$ respectively. For convenience, in the following we will use $(-A)$ to denote the minimal closed extension of the two-dimensional negative Laplacian defined on twice continuously differentiable complex valued functions on $D$ which vanish on $\partial D$ and $D(A) \subset H$.

It is known in linear theory of elasticity that the transverse displacements of a thin isotropic vibrating plate are governed by (1.1) where $D(A) \subset C^4(D)$ and the

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elements of $D(A)$ satisfy either of the following boundary conditions

$$u|_{\partial D} = 0 \quad \text{and} \quad \left. \frac{\partial u}{\partial n} \right|_{\partial D} = 0 \quad (1.4)$$

or

$$u|_{\partial D} = 0 \quad \text{and} \quad \left[ \Delta u - \frac{1 - \sigma}{\rho} \frac{\partial u}{\partial n} \right]_{\partial D} = 0 \quad (1.5)$$

where $\partial/\partial n$ represents the outward normal derivative on $\partial D$, $\rho$ is the radius of curvature of $\partial D$ and $\sigma$ is the Poisson's ratio of the plate material. It is assumed that $\sigma$ is constant and $\sigma > 0$.

If there exists an operator $B$ such that $A = B^2$, then we can introduce the following definitions (see Hille and Phillips [1]).

**Definition 1.** A mapping $u$ defined on $J$ with values in $H$ is said to be a solution of the problem (1.1)–(1.2) if it is twice strongly continuously differentiable on $J$, $u \in D(A)$, $du/dt \in D(B)$ for every $t \in J$, $Au$ and $B(du/dt)$ are strongly continuous in $t$ on $J$ and $u(\cdot)$ satisfies (1.1) and (1.2).

**Definition 2.** An operator $B$ is said to have the property $C$ if i) $B$ is a closed linear operator with domain $D(B)$ dense in $H$, ii) the spectrum of $B$ lies on the imaginary axis, iii) the resolvent of $B$ satisfies

$$\| R(\lambda, B) \| \leq \frac{C}{|\lambda| + 1}, \quad -\infty < \lambda < \infty \quad (1.6)$$

for some positive constant $C$ depending on $B$.

If the conditions of definition 2 are satisfied then it will follow that the operator $B$ is the infinitesimal generator of a group of bounded linear operators $T(t)$, $-\infty < t < \infty$ on $H$ and furthermore that

$$\| T(t) \| \leq Ce^{-\lambda t} \quad -\infty < t < \infty \quad (1.7)$$

holds.

In the following, by a plate operator we will mean the operator $A$ defined by (1.3) the elements of whose domain satisfy either of the boundary conditions (1.4) or (1.5). The following two lemmas are fundamental for our result.

**Lemma 1.** The operator $A$ defined by (1.3) along with either of the boundary conditions (1.4) or (1.5) is self-adjoint and there exists a constant $\beta > 0$ such that

$$\langle Au, u \rangle \geq \beta^2 \| u \|^2 \quad u \in D(A) \subset H. \quad (1.8)$$

**Proof:** For details of a proof of the assertion we refer to Miklin [2].