Constrained Control for Uncertain Linear Systems

F. Blanchini

Communicated by G. Leitmann

Abstract. The linear state feedback synthesis problem for uncertain linear systems with state and control constraints is considered. We assume that the uncertainties are present in both the state and input matrices and they are bounded. The main goal is to find a linear control law ensuring that both state and input constraints are fulfilled at each time. The problem is solved by confining the state within a compact and convex positively invariant set contained in the allowable state region.

It is shown that, if the controls, the state, and the uncertainties are subject to linear inequality constraints and if a candidate compact and convex polyhedral set is assigned, a feedback matrix ensuring that this region is positively invariant for the closed-loop system is found as a solution of a set of linear inequalities for both continuous and discrete time design problems.

These results are extended to the case in which additive disturbances are present. The relationship between positive invariance and system stability is investigated and conditions for the existence of positively invariant regions of the polyhedral type are given.

Key Words. Stabilization, uncertain systems, constrained control, positive invariance, linear programming.

1. Introduction

In practical application of control theory, some difficulties often arise due to imperfect knowledge of the dynamical model. For this reason, the control of uncertain systems is currently of great interest and many recent contributions are found in the literature.

1The author is grateful to Drs. Vito Cerone and Roberto Tempo for their comments.
2Researcher, Dipartimento di Matematica ed Informatica, Università di Udine, Udine, Italy.
As it is known, many important results in this subject are based on a Lyapunov function approach (Refs. 1–4). The fundamental idea is to compute the control law such that the resulting closed-loop system admits an assigned function as a Lyapunov one. The so-called matching conditions are a fundamental problem related to these techniques. This question is reconsidered in Ref. 5, where a mismatch measure is introduced and a stabilizing control is assured if the mismatch is sufficiently small.

Recent developments are found in Refs. 6–8 in which conditions for quadratic stabilizability are given; namely, the candidate Lyapunov function is chosen as a quadratic positive one. A Riccati equation approach is presented in Refs. 9 and 10.

It is well known that a crucial point in control engineering is the often unavoidable presence of bounds on the state and the control variables. Broadly speaking, the control law must be computed such that the state vector of the closed-loop system is confined in a compact region, named the allowable state region, while the control signal does not violate the constraints thus resulting in saturations. We refer to this problem as the constrained regulator problem (CRP).

The linear constrained control problem for linear discrete-time systems is considered in Refs. 11–13. It is shown that the concept of a positively invariant set is a fundamental one for the solution of the problem. Necessary and sufficient conditions for the positive invariance of a polyhedral set for discrete-time systems are given in Refs. 14 and 15.

If input constraints are present, the positive variance conditions must be considered together with the admissibility one (see Ref. 16); that is, the control signal corresponding to each state must fulfill the constraints.

The constrained control problem for discrete-time linear systems is considered in Ref. 17, where linear programming techniques of on-line control computation described in Refs. 18–20 are extended to systems with additive disturbances. The linear state feedback in the presence of additive noise is considered in Ref. 21.

In the present paper, we consider the linear constrained regulator problem (LCRP) for uncertain linear systems. We assume that time-varying bounded uncertainties lie in both state and input matrix.

In Section 2, some basic definitions and results are introduced. It is proven that the LCRP has a solution provided that an initial state set contained in the allowable state region which is positively invariant and admissible for the closed-loop system is found. Some conditions equivalent to the positive invariance for compact sets are given.

In the following sections, polyhedral bounds for state, control, and uncertainties are considered. It is shown that the feedback matrix may be obtained as a solution of a set of linear inequalities. No matching assumption is made.