where \((,\) denotes the scalar product in the space \(\mathbb{R}^8\) with the metric \(\gamma_0\). Direct computation shows that it is possible to construct a total of seven continuous vectors which are mutually orthogonal and tangent to the sphere \(G_7\) in the metric of the space \(\mathbb{R}^8\). In the equation for \(\frac{d\psi}{d\tau}\) only the first four vectors enter which, apparently, admits a dual interpretation. Vectors of the tangent space to the space of the outer coordinates \(x^\mu\) are to be identified with tangent vectors to the space of the inner variables. Since the latter is seven-dimensional, only a four-frame of the tangent bundle of the space of inner variables is identified with the tangent vectors to the space of outer coordinates. On the other hand, two of the three vectors not entering the equation for \(\frac{d\psi}{d\tau}\), which are tangent to the "sphere" \(G_7\) and orthogonal to the vectors \(i\gamma^\mu\phi\), do not commute with the matrix \(i\) defining the symplectic structure on the space of inner variables and hence do not preserve the canonical dynamics, while inclusion of the last of these in the equation for \(\frac{d\psi}{d\tau}\) leads to a violation of invariance of the equation both under time inversion \(x^0 \rightarrow -x^0\) and under inversion of the spatial coordinates \(x^1, x^2, x^3 \rightarrow -x^1, -x^2, -x^3\).

In conclusion, the authors wish to thank Yu. G. Borisovich for his constant interest in this work.

LITERATURE CITED


RADIATION OF AN ELECTRON IN AN ELECTRIC FIELD. II

N. I. Fedosov and G. I. Flesher

By the methods of quantum electrodynamics, the radiation of an electron moving in the field of a traveling electric wave is considered. It is shown that, in contrast to the classical case, the result given by quantum theory for the spectral-angular distribution of the radiant energy differs from the corresponding expression for an electron in a constant homogeneous electric field, even in the terms proportional to Planck's constant.

In [1] it was shown by the methods of classical electrodynamics that the radiation of an electron in the field of a longitudinal electric wave is similar to the radiation in a constant homogeneous field of intensity \(E = E(\xi')(\xi = ct - z)\), where \(\xi'\) is the point at which the particle velocity becomes equal to the velocity of light.

In the present work, we consider the same problem by the methods of quantum electrodynamics. Since the electric field leads to the creation of electron-positron pairs from a vacuum, it is necessary to find the correct form for the matrix elements of quantum processes in a strong external electromagnetic field. This problem was evidently first examined in [5], where formulas for the calculation of the probability of quantum processes in pair-producing fields were given. However, for our present purposes, direct application of the formulas obtained in [5] is not possible, because the solutions of the Dirac equation in the field of a longitudinal electric wave obtained in [3] are not normalized in the plane \(t = \text{const}\). However, these solutions form a complete and orthonormalized system of functions in the zero plane. As a result, it is necessary to consider how the probability of photon emission in a strong external electromagnetic field should be written in zero-plane formalism. In particular, we obtain in this way a probability of photon emission by an electron which only in certain special cases leads to the corresponding expression used in [2] for the calculation of the radiation in a constant homogeneous electric field.

§1. Probability of Photon Emission by an Electron in Zero-Plane Formalism

We introduce the nonorthogonal coordinate system \{u^\mu\}

\[
\begin{align*}
  u^0 &= \frac{1}{\sqrt{2}} (x^0 - x^3), \\
  u^1 &= x^1, \\
  u^2 &= x^2, \\
  u^3 &= \frac{1}{\sqrt{2}} (x^0 + x^3).
\end{align*}
\]

The components of an arbitrary vector \(a\) with respect to the new coordinate system will be denoted in the usual way by \(a^\mu\).

In zero-plane formalism [4], the Hamiltonian of the system consisting of interacting electron-positron and electromagnetic fields takes the form

\[
H = H_0 + H_1,
\]

where

\[
H_0 = \frac{\hbar^2}{2m} \psi_{(+)}(u) \psi_{(+)}(u) A_0(u) + 2^{-\sigma} \phi_{(+)}(u)(\gamma^j \gamma_{+}^j + \nu_3) \psi_{(+)}(u) \gamma^0 \gamma_{+}^0 \psi_{(+)}(u) \psi_{(+)}(u),
\]

\[
H_1 = \frac{\hbar^2}{2m} \psi_{(+)}(u) \psi_{(+)}(u) A_0(u) + 2^{-\sigma} \phi_{(+)}(u)(\gamma^j \gamma_{+}^j + \nu_3) \gamma^j A_j(u) \psi_{(+)}(u) \gamma^0 \gamma_{+}^0 \psi_{(+)}(u) - eA_j(u) \psi_{(+)}(u) \gamma^j \gamma^0 \gamma_{+}^j \gamma_{+}^0 \psi_{(+)}(u), \quad j = 1, 2.
\]

Here \(A_\mu(u)\) is the potential of the external field; \(A_\mu\) is the potential operator of the quantum electromagnetic field; \(H_{\text{ph}}\) is the Hamiltonian of the electromagnetic field; \(\psi_{(+)}(u)\) is related to the solution of the Dirac equation as follows

\[
\psi_{(+)}(u) = \rho_{(+)}(u), \quad \rho_{(+)} = \frac{1}{2} \gamma^0 \gamma^3 = \frac{1}{2} (1 - \gamma_3),
\]

\[
p_j = i\gamma_j - eA_j(u); \quad \mu_0 \text{ is the electron mass.}
\]

The effect of applying the operator \(P_3^{-1}\) is given by the expression

\[
2i P_3^{-1} \phi(u) = \int_{-\infty}^{+\infty} d\xi z(u^3 - z) \exp \left\{ -i e \int_{\xi}^{u} A_3(u_\perp, u^0, \xi') d\xi' \right\} \exp \left\{ -i e \int_{\xi}^{u} A_3(u_\perp, u^0, \xi') d\xi' \right\} \phi(u_\perp, u^0, \xi).
\]

From Eq. (1) it is evident that \(H_0 = H_{\text{Me}} + H_{\text{ph}}\), where \(H_{\text{Me}}\) is the density of the Hamiltonian of the electron-positron field with respect to the external electromagnetic field. \(H_1\) is the density of the interaction Hamiltonian.

Following the procedure outlined in [5], we choose the external electromagnetic field in the form

\[
\tilde{A}_\nu(u) = \begin{cases} 
  A_\nu(u) & -\infty < u^0 \leq u^0_0 \\
  A_\nu(u) & u^0_0 \leq u^0 \leq u^0_1 \\
  A_\nu(u) & u^0_1 < u^0 < +\infty,
\end{cases}
\]

We assume that the fields \(\tilde{A}_\mu\) and \(A_\mu\) do not produce pairs, i.e., in these fields the solutions may be classified according to their particle-antiparticle character.

The change of state in "time" \(u^0\) is given by the evolution operator

\[
V(u^0, u^0_0) = T \exp \left\{ -i \int_{u^0_0}^{u^0} H(u) du \right\}.
\]

If we choose the evolution operator in the form

\[
V(u^0, u^0_0) = U(u^0, u^0_0) S(u^0, u^0_0) U^{-1}(u^0, u^0_0),
\]

then, substituting this expression into the Schrödinger equation with the Hamiltonian in Eq. (1), the following equations for determining \(U\) and \(S\) may be obtained:

\[
\begin{align*}
  &i \frac{\partial U(u^0, u^0_0)}{\partial u^0} = H_0(u) U(u^0, u^0_0), \\
  &i \frac{\partial S(u^0, u^0_0)}{\partial u^0} = \tilde{H}_1(u) S(u^0, u^0_0).
\end{align*}
\]

\[
\tilde{H}_1(u) = U^{-1}(u^0, u^0_0) H_1(u) U(u^0, u^0_0).
\]

761