THE RADIATIVE RELAXATION TIME IN
THE CHROMOSPHERE

R. G. GIOVANELLI
National Measurement Laboratory, CSIRO, Sydney, Australia 2070

(Received 27 December, 1977; in revised form 25 July, 1978)

Abstract. The cooling effect of emission in the spectral lines, which dominates over continuous emission in the chromosphere and becomes important first around the temperature minimum, modifies greatly the radiative relaxation time \( t_r \) in the solar atmosphere. This rises from low photospheric values to a maximum of \(-8\) min just above \( T_{\text{min}} \), falls in the low chromosphere to \(-1.5\) min because of line emission, but rises again to \(-6\) min at \( T \approx 7000-8400 \) K in the chromosphere where hydrogen ionization increases the specific heat.

1. Introduction

Radiative cooling of the upper photosphere and chromosphere enters into at least two problems: the steady-state temperature distribution, and temperature fluctuations in compressive waves. The former has been discussed by Jordan (1977), Athay (1966), and others, while Athay (1976) summarizes compressional effects.

The characteristic radiative relaxation (or cooling) time, \( t_r \), increases from \( \approx 1 \) s to \( \approx 2 \) min in the range of continuum optical depths \( \tau_c = 1 \) to \( 10^{-3} \) (Athay 1976, p. 274). It is generally believed that \( t_r \) continues to increase monotonically outwards, e.g. to over \( 10 \) min at \( \tau_c = 10^{-5} \) (Stix, 1970; Levy, 1974) where acoustic waves of period \( \approx 5 \) min would be effectively adiabatic. We shall see below that this is not the full story. The relaxation time has a maximum of \( \approx 8 \) min just above the temperature minimum \( T_{\text{min}} \), above which it falls quickly to \( \approx 1.5 \) min before rising again when the specific heat increases due to the ionization of hydrogen.

For emission mainly in the continuum, Spiegel (1957) gave an expression for \( t_r \), which, when the perturbation is thin optically, reduces to

\[
 t_r = \frac{\rho C_v}{16 \kappa c \sigma T^3}, \tag{1.1}
\]

where \( \rho \) is the density, \( C_v \) the specific heat (of unit mass) at constant volume, \( \kappa_c \) the mean absorption coefficient, and \( \sigma \) the Stefan–Boltzmann constant. Athay (1966) subsequently discussed radiative loss mechanisms, concluding that the H\(^+\) continuum dominates in the low chromosphere out to \( \tau_c \approx 10^{-5} \) (\( T \approx 5280 \) K).

The problem is complicated by the difficulties of allowing properly for self-absorption in the spectral lines, and the result obtained is sensitive to the assumptions. In turn, \( t_r \) its variation throughout the chromosphere, and its use in deriving wave properties from observed intensity-velocity phase relations are also dependent on those assumptions.
The following analysis is approximate but finds much greater net emission of line and recombination radiation than of \( H^- \) continuum above \( \tau_e \approx 10^{-4} \). This causes a big reduction in relaxation times there as compared with values obtained with solely \( H^- \) continuum radiation. The results are summarized in Table VII.

### 2. The Energy Equation

The rate of change of the heat content of a mass \( M \) of gas containing \( n_A \) atoms, under the combined influences of radiation, expansion and ionization, is given by

\[
\frac{d}{dt} \left[ \frac{3}{2} (n_A + n_e) kT \right] = (Q - E) V - \rho \frac{dV}{dt} - \chi^i \frac{dn_e}{dt} + W, \tag{2.1}
\]

where \( Q \) and \( E \) are the rates of absorption and emission of radiation per unit volume, \( V \) is the volume, \( \rho \) the pressure, \( \chi^i \) the ionization potential, and \( n_e \) the total content of free electrons. In addition we have allowed for unspecified heating at a rate \( W \) which is used to balance \( (Q - E) V \) in the unperturbed atmosphere; \( W \) is taken to be a local constant and disappears ultimately from the analysis. The ionization term is significant only at temperatures well above \( T_{\min} \) where hydrogen is being ionized. (2.1) may be rearranged as

\[
W = \frac{3}{2} k \frac{d}{dt} \left[ (n_A + n_e) T \right] + \rho \frac{dV}{dt} + \chi^i \frac{dn_e}{dt}. \tag{2.2}
\]

For a perturbation where the temperature exceeds the undisturbed value (at which \( (Q - E) V + W = 0 \)) by \( \Delta T \), the radiative relaxation time may be defined by

\[
t_r = \frac{\Delta T}{d T / d t}. \tag{2.3}
\]

As in sound waves, the thermal changes take place under conditions of constant pressure.

We shall now derive an equation similar to (1.1) in order to see the approximations involved. First, ionization changes are neglected, whence (2.2) simplifies to

\[
(Q - E) V + W = C_p M \frac{d(T)}{dt}, \tag{2.4}
\]

where \( C_p \) is the usual specific heat at constant pressure. But

\[
\Delta T \{ (Q - E) V + W \} = \frac{d \{ (Q - E) V \}}{dT} \Delta T. \tag{2.5}
\]

Then (2.3), (2.4) and (2.5) lead to

\[
t_r = - \frac{C_p M}{d \{ (Q - E) V \} / d T}. \tag{2.6}
\]

Next assume that \( \kappa_e \) is independent of wavelength. Then if emission and absorption