THE ENERGY DISTRIBUTION OF MEDIUM-ENERGY SECONDARY ELECTRONS IN THE PHENOMENON OF SECONDARY ELECTRON EMISSION FROM A METAL

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The transport process of medium-energy secondary electrons in a metal is investigated on the assumption that they are scattered by a screening Coulomb potential. Scattering angles are selected which satisfy the relation \( E' \sin^2 \theta < E_0 \). In this case the integral equations for the expansion coefficients of the distribution function of the secondary electrons reduce to linear differential equations whose solution is carried out on a computer. The energy distribution of the outer secondary electrons is derived for the following two cases: 1) the mean free path \( l(E) \) is constant; 2) \( l(E) \) depends on the energy according to Ritchey and Ashley [6]. Comparison with experiment shows that the distribution of the secondary electrons is closer to the experimental one in the first case.

The scattering of secondary electrons in the energy region which is important to the phenomenon of secondary electron emission occurs mainly due to collisions with the conduction electrons of a metal. The interaction potential between a secondary electron and the electrons of the metal, which are considered to be free, taking account of the screening by the collective motion of the electron gas, is, according to [1], of the form:

\[
\varphi (r) = \frac{e}{sr} e^{-qr}
\]  

(1)

where \( q = 1/r_0 \) and \( r_0 = \left[ \frac{\hbar^2}{4m_e^2(\pi/3n)^{1/2}} \right]^{1/2} \) is the screening radius in the case of a degenerate electron gas.

In this case the effective scattering cross section in the system of the center of mass of both particles is defined by the equation

\[
\sigma (\theta) = \left[ \frac{e^2/q}{mv_0^2 (1 - \cos \theta) + \hbar^2 q^2/2m} \right]^2.
\]  

(2)

In connection with the geometry adopted in the literature [2, 3] for the transport process, it is necessary to transform to the laboratory system of coordinates in Eq. (2)

\[
\sigma (\theta) = \left[ \frac{e^2/q}{mv_0^2 \sin^2 \theta + \hbar^2 q^2/2m} \right]^2 \cos \theta,
\]  

(3)

where \( \theta \) is the scattering angle of a secondary electron which has the velocity \( v' \) prior to collision with an electron of the metal.

In the case of low energies and small scattering angles Eq. (3) reduces to spherically-symmetric scattering in the center-of-mass system, which was taken into consideration by Wolff [2] in his discussion of the transport of secondary electrons in a metal.

In this paper the transport process of medium-energy (\( E_{\text{max}} = 100 \text{ eV} \)) secondary electrons whose scattering angles satisfy the relationship

\[
E' \sin^2 \theta \ll E_0, \text{where} \ E_0 = \frac{\hbar^2 q^2}{2m}.
\]  

(4)

is investigated.
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<th>$E, \text{eV}$</th>
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<th>$\Psi_2 (E)$</th>
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In this case the scattering function $F(E', E', \theta)$ with the cascade process taken into account is represented as

$$F(E', E', \theta) = \frac{4E_0}{\pi} \frac{\cos \theta}{(E' \sin^2 \theta + E_0)^2} \Theta (E_0 - E' \sin^2 \theta),$$

where the step function

$$\Theta (E_0 - E' \sin^2 \theta) = \begin{cases} 1 & \text{for } E' \sin^2 \theta \leq E_0 \\ 0 & \text{for } E' \sin^2 \theta > E_0 \end{cases}$$

has been used.

In order to determine the energy distribution of the medium-energy secondary electrons, it is necessary to solve Boltzmann's equation, which in the case of the perpendicular incidence of the primary electron beam on a metal–vacuum interface is, according to [2], written in the following form:

$$\frac{\partial N(E, \beta)}{\partial t} = S(E, \beta) + 2\pi \int_0^\infty \frac{\partial' N(E', \beta')}{\partial t} F(E', E, \theta) dE' \sin \beta' d\beta'.$$

Here the notation is the same as in the papers [2, 3]. In our case $F(E', E, \theta)$ has the form of Eq. (5).

Introducing the notation $\nu N(E, \beta) / l(E) = \psi(E, \beta)$ and resolving the functions $\psi(E, \beta)$, $F(E', E, \theta)$, and $S(E, \beta)$ into series in Legendre polynomials, we obtain the following equations for the expansion coefficients $\psi_l(E)$:

$$\psi_l(E) = S_l(E) + \int \psi_l(E') F_l(E', E) dE' \quad (l = 0, 1, 2, ...),$$

where

$$F_0(E', E) = \frac{2}{E'};$$

$$F_1(E', E) = \frac{4E_0}{E^2} \left[ E' \left( 1 - \frac{1}{2} \sqrt{1 - \frac{E_0}{E'}} \right) - \frac{1}{E} \ln \frac{(V' E' + E_0 + V E)}{E_0 V \frac{1}{2}} \right];$$

$$F_2(E', E) = \frac{2}{E'} \left[ 1 - 3 (\ln 2 - 1) \frac{E_0^2}{E} \right];$$

$$F_3(E', E) = \frac{10E_0}{E^2} \left( 3 - 2.5 \sqrt{1 - \frac{E_0}{E'}} \right) + \frac{8}{E} \left( 2^{2} - 1 - \frac{E_0}{E'} \right);$$

$$- \frac{E_0}{E^2} 24 + 30 \frac{E_0}{E'} \ln \frac{(V' E' + E_0 + V E)}{E_0 V \frac{1}{2}}.$$