BRITTLE FRACTURE PROPAGATION IN ROCK UNDER COMPRESSION

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ABSTRACT

The results of studies of the initiation and propagation of fracture from a single Griffith crack in a biaxial compressive stress field are reported. It is concluded that Griffith's theory of brittle fracture offers a reliable prediction of the fracture initiation stress but that the resulting fracture propagation from a single crack cannot account for the macroscopic fracture of a specimen. Some preliminary results of studies on crack arrays and on the effects of crack closure in compression are presented. The applicability of these results to the prediction of rock fracture in predominantly compressive stress fields is discussed.

INTRODUCTION

One of the most serious problems encountered in deep-level gold mining in South Africa is the sudden and violent fracture of rock, known in the mining industry as a rockburst. Seismic location of the foci of these rockbursts has established that they occur most frequently in the zones of high compressive stress which surround the working faces of the mining excavations. Since the mining industry is constantly striving to minimize the hazards created by these rockbursts, an understanding of the mechanism of rock fracture under compressive stress conditions is of vital interest.

Previous research has shown that Griffith's brittle fracture theory, modified to account for the effects of crack closure in compression, is a useful basis for the study of the fracture of hard rock. Brace, in discussing the nature of the pre-existing cracks in rock, suggests that the grain boundaries act as or contain micro-cracks while joints and faults can be regarded as macro-cracks.

An analysis of the stress distribution around a crack indicates the points of fracture initiation as well as the initial direction of crack propagation. As a result of the change in stress distribution associated with fracture propagation it is, however, impossible to predict the final path of the propagating crack. Consequently, a serious limitation of the Griffith theory lies in the fact that it can only be used to predict fracture initiation.

In its usual form, it yields no information on the rate or direction of fracture propagation.

In studying the fracture of brittle materials subjected to tension, fracture is normally expected in a direction perpendicular to the applied tension, in other words, in the plane of the critically oriented crack. In the case of a brittle material subjected to compressive stress, one might therefore expect that fracture propagation will also follow the direction of the most critically oriented crack, i.e. the one which is inclined at 20-30° to the major principal field-stress direction. It will be shown in this paper that this anticipated result is incorrect and that there is no simple relationship between the critical orientation of the original "Griffith crack" and the orientation of the macroscopic fracture surface of a specimen.

THEORETICAL CONDITIONS FOR FRACTURE INITIATION

Griffith's original postulate on fracture initiation was based on energy considerations and his equations contained a surface energy term. Because


** Superscripts refer to the list of references at the end of this paper.
of the difficulty of evaluating experimentally the surface energy of a material, an alternative approach, which considers the stress concentration at the crack tip, has been adopted by most workers in rock mechanics.

The current interpretation (7) of Griffith's theory is that fracture initiates when tensile stress induced at or near the tip of an inherent crack exceeds the molecular cohesive strength of the material. Since the molecular cohesive strength is difficult to determine by direct measurement, the fracture criterion is expressed in terms of the uniaxial tensile strength of the material (3).

In order that the reader may readily follow the equations which are used in this paper, a brief derivation of these equations, based upon the work by Griffith (4) and McClintock and Walsh (5), follows.

It is assumed that the crack from which the fracture of a brittle rock originates can be regarded as a flat elliptical opening in a two-dimensional body which is subjected to a stress system* as illustrated in Figure 1.

![Figure 1. Stresses acting upon a crack which is inclined at an angle $\psi$ to the direction of the major principal stress $\sigma_1$.](image)

The stress field around an elliptical opening is related to the elliptical coordinates $\xi$ and $\eta$ which are defined by the following equations of transformation of a rectangular system of coordinates $x$ and $z$:

$$x = c \sinh \xi \sin \eta,$$
$$z = c \cosh \xi \cos \eta$$

In Figure 1, the system of rectangular coordinates $x$, $z$ is parallel to the axes of the elliptical opening; it is inclined at an angle $\psi$ with respect to the system of rectangular coordinates $x'$, $z'$ which is parallel to the directions of the principal stresses $\sigma_1$ and $\sigma_3$. Of these, $\sigma_1$ is algebraically largest and $\sigma_3$ algebraically smallest of the three principal stresses.**

* Because of the predominance of compressive stress in rock mechanics problems, compressive stress is taken as positive.

** In this analysis, the intermediate principal stress $\sigma_3$ is assumed to have a negligible influence upon fracture.