After quite involved but standard estimates the details of which we omit, using the assumptions of the theorem and Hölder's inequality (10) will hold for all \( r > 1 \) if we note that for any left quasicontinuous \( (\mathcal{F}, \mathbb{P}) \)-local martingale \( \bar{L}_t, \ t > 0, \bar{L}_0 = 0 \) the process

\[
\bar{L}_t = \exp \left\{ r L_t - \frac{1}{2} r^2 \langle L^r \rangle_t - \int_0^t (e^{r s} - 1 - r s) \Pi^r (ds, dx) \right\}, \ t > 0,
\]

is a \( (\bar{\mathcal{F}}, \mathbb{P}) \)supermartingale and \( \mathcal{E}_t \bar{L}^r (r) \leq 1, \ t > 0 \). The validity of the last assertion is easily verified by calculating the stochastic differential of the process \( \bar{L}(r) \). The theorem is proved.

**LITERATURE CITED**


**DISTRIBUTION OF DISTANCE BETWEEN POINTS OF TWO CONGRUENT CONVEX DOMAINS**

E. Gešiauskas

The goal of the paper is to express the distribution function of the distance in terms of the simplest integral.

**THEOREM.** The distribution function of the distance \( R \) between points of two congruent convex domains, moved apart parallel by the distance \( Z \), is equal to

\[
P (R \leq x) = 2F^{-1} \int [r \sigma (\varphi, \rho) - r^2] [1 - 2x R^{-2} \cos^2 \varphi]^p \sigma (\varphi, \rho) d\rho d\varphi,
\]

\( s \leq R \leq x, \ \sigma (\varphi, \rho) = (R \sin \varphi^* - \rho)/R \cos \varphi^*, \ \sigma_1 \leq \varphi^* \leq \sigma_2, \)

where \( R^2 = z^2 + r^2 + 2r \sin \varphi, \ r \) is the distance from the point \( P_2 \) to the image \( P_1 \) of the point \( P_1 \) in the domain \( K_2, \ \varphi \) is the direction of the perpendicular \( p \), dropped from the origin to the line \( G \), passing through the points \( P_1 \) and \( P_2 \), where \( \sigma (\varphi, \rho) \) is the length of the chord on the line \( G, \ \varphi^* \) is the direction of the perpendicular \( p^* \) to \( R \).

**Proof.** We take two congruent convex domains, moved apart by parallel translation by the distance \( z \). The distribution function of the distance between the points of these domains is equal to
\[ P(R \leq x) = \mu(s \leq R \leq \infty | P_1, P_2) \mu(s \leq R \leq T | P_1, P_2) = \mu(s \leq R \leq x | P_1, P_2) / F^2 = \mu(x) / F^2, \]

where \( \mu \) is the measure of the set of pairs of points, \( s \) is the smallest distance, and \( T \) is the largest distance between points of the domains \( K_1 \) and \( K_2 \).

\[ \mu(x) = 2 \int_{s \leq R \leq x} RdG^* d\varphi^* dR, \]

where \( dG^* \) is the density of the set of lines \( G^* \), joining points \( P_1 \) and \( P_2 \), \( t^* \) is the distance of the point \( P_2 \) from the base of the perpendicular \( p^* \), dropped from the origin to the line \( G^* \). \( dG^* = dp^* d\varphi^* \), where \( \varphi^* \) is the direction of the perpendicular \( p^* \).

\[ \mu(x) = 2 \int_{s \leq R \leq x, \varphi \in \varphi^*} Rd\varphi^* dR. \]  

From the triangle, whose vertices are the points \( P_1 \), \( P_2 \) and the image of the point \( P_2 \) in the domain \( K_2 \), and sides are \( |P_1P_2| = R \), \( |P_1P_1'| = r \), and \( |P_1P_2| = r \), we have that

\[ \frac{R}{\sin(3\pi/2 - \varphi)} = \frac{z}{\sin(\varphi - \varphi^*)} = \frac{r}{\sin(r - \pi/2)}, \]

where \( \varphi \) is the direction of the perpendicular \( p \) from the origin to the line \( G \), passing through the points \( P_1 \) and \( P_2 \), and the direction of transport of the domain coincides with the direction \( \varphi^* = 0 \).

For the same triangle one has

\[ R^2 = z^2 + r^2 - 2zr \cos(3\pi/2 - \varphi) = z^2 + r^2 + 2zr \sin \varphi. \]  

We express \( dR, dp^*, dp^*, di^* \) in terms of \( dr, dp, d\varphi, dt \).

\[ dR = R^{-1} \left[ (r + z \sin \varphi) dr + zr \cos \varphi d\varphi \right]. \]

\[ d\varphi^* = r \sin(\varphi - \varphi^*) \]

\[ z \sin \varphi^* d\varphi^* = \sin(\varphi - \varphi^*) dr + r \cos(\varphi - \varphi^*)(d\varphi - d\varphi^*), \]

\[ [z \sin \varphi^* + r \cos(\varphi - \varphi^*)] d\varphi^* = \sin(\varphi - \varphi^*) dr + r \cos(\varphi - \varphi^*) d\varphi. \]

We make use of the equalities

\[ \sin(\varphi - \varphi^*) = -R^{-1} z \cos \varphi, \]  

\[ \cos(\varphi - \varphi^*) = R^{-1}(r + z \sin \varphi), \]  

\[ \cos \varphi^* = R^{-1} r \cos \varphi, \]

\[ \sin \varphi^* = R^{-1}(z + r \sin \varphi). \]

\[ [R^{-1} z (z + r \sin \varphi) + R^{-1} r (r + z \sin \varphi)] d\varphi^* = -R^{-1} z \cos \varphi dr + R^{-1} r (r + z \sin \varphi) d\varphi, \]

\[ d\varphi^* = R^{-2} [-z \cos \varphi dr + (r + z \sin \varphi) d\varphi]. \]

According to the rules for the exterior product of differential forms (cf. [2, p. 13]), we get that

\[ dRd\varphi^* = R^{-3} [r (r + z \sin \varphi)^2 - rz^2 \cos^3 \varphi] d\varphi. \]

One has the relations

\[ p^*/[t - p \tan(\varphi - \varphi^*)] = \sin(\varphi - \varphi^*), \]

\[ r^*/[t - p \tan(\varphi - \varphi^*)] = \cos(\varphi - \varphi^*). \]

Whence

\[ p^* = t \sin(\varphi - \varphi^*) + p \cos(\varphi - \varphi^*), \]  

\[ r^* = t \cos(\varphi - \varphi^*) - p \sin(\varphi - \varphi^*). \]  

From (7), (8), and (4), (5), we get

\[ p^* = R^{-1} p (r + z \sin \varphi) - R^{-1} tz \cos \varphi, \]

\[ t^* = t \cos(\varphi - \varphi^*) - p \sin(\varphi - \varphi^*). \]