Two-dimensional model problems for the equations of small oscillations of a rotating fluid have the form

\[
\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial y^2} + \frac{n^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial y^2} = 0,
\]

(1)

\[u|_{t=0} = 0,
\]

(2)

\[u_t|_{t=0} = u_0(x, y), \quad \frac{\partial u}{\partial t}|_{t=0} = u_1(x, y),
\]

(3)

where \(\Gamma\) is the boundary of the bounded convex domain \(\Omega\) in the \(x, y\) plane. Assume that \(\Gamma\) has the equation in polar coordinates \(x = x(\phi), y = y(\phi)\). Assume that \(x(\phi), y(\phi)\) are thrice continuously differentiable functions and that

\[
\frac{x'y^2 - x^2 y'}{x^2 + y^2} \leq q < 0.
\]

Denote by \((\Delta - n^2)^{-1}f = u\) the solution of the problem

\[(\Delta - n)u = f, \quad u|_{\Gamma} = 0.\]

In Sobolev space \(W^2_2(\Omega)\) with scalar product

\[
[u, v] = \int \left( \frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial y} + n^2 u v \right) dx dy,
\]

where the operator \(A_n = -(\Delta - n^2)^{-1}(\partial^2 / \partial y^2)\) is self-adjoint and bounded, the problem (1)-(3) is equivalent to the equation

\[
\frac{\partial^2 u}{\partial t^2} = A_n u.
\]

The operator \(A_n\) admits bounded extension into the whole of \(L^2(\Omega)\). The generalized eigenfunctions of \(A_n\), belonging to \(L^2(\Omega)\), are generalized solutions of the problem

\[
- \lambda^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \lambda^2 n^2 u = 0,
\]

(4)

\[u|_{\Gamma} = 0.
\]

Along with problem (4)-(5), we shall investigate the corresponding inhomogeneous problem

\[
- \lambda^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \lambda^2 n^2 u = f,
\]

(4')

\[u|_{\Gamma} = 0.
\]

In the present article we construct, for certain values of the parameter \(\lambda\), all the solutions belonging to \(L^2(\Omega)\) of problems (4)-(5) and (4')-(5'), and we also examine the smoothness of the inhomogeneous problem (4')-(5').

Certain results of [1-3], which will be required later, will next be quoted. Let \(\Gamma\) be the circle \(x = \cos \varphi, y = \sin \varphi\). The straight line \(x - \cot \varphi = x(\varphi) - \cot \varphi(\varphi)\) cuts \(\Gamma\) at the point \(\psi\), distinct from \(\varphi\), given by

\[
\psi = - \varphi + 2\pi - \pi.
\]
A continuous function \( f_1(\varphi, \alpha) \) is constructed in [1], which, as in the case \( x = \cos \varphi, y = \sin \varphi \), has the property
\[
xf_1(\varphi, \alpha) - \cot \alpha y(f_1(\varphi, \alpha)) = x(\varphi) - \cot \alpha y(\varphi).
\]
We put \( f_0(\varphi, \alpha) = \varphi \), and for all \( n \geq 1, f_n(\varphi, \alpha) = f_1(f_{n-1}(\varphi, (-1)^{n+1}\alpha)). \)

The step line whose corners are the points of \( \Gamma \) with coordinates \( x(f_k(\varphi, \alpha)), y(f_k(\varphi, \alpha)) \) where \( \varphi, \alpha \) are fixed, and which is formed by segments of the straight lines
\[
x(f_k(\varphi, \alpha)) + (1)^{k+1}y(f_k(\varphi, \alpha)) \cot \alpha = x + (1)^{k+1}y \cot \alpha,
\]
lying inside \( \Omega \), will be referred to as the trajectory issuing from the point \( \varphi \) at the angle \( \alpha \). If the trajectory is closed, i.e., if for certain \( k \) and \( l \) we have \( f_k(\varphi, \alpha) = \varphi + 2\pi l_1, f_k(\varphi, -\alpha) = \varphi + 2\pi l_2 \), we shall refer to it as a cycle with \( 2k \) links.

The following was proved in [1]:

**Lemma 1.** For every \( N \) and point \( \varphi \) on the boundary \( \Gamma \) there exist \( \alpha_N(\varphi) \) \((l = 0, 1, \ldots, 2N - 1)\) such that the trajectory issuing from the point \( \varphi \) at the angle \( \alpha_N(\varphi) \) is a cycle with \( 2N \) links. The function \( \alpha_N(\varphi) \) is a twice continuously differentiable periodic function of \( \varphi \).

We put \( \lambda_{n,k}(\varphi) = \cot \alpha_N(\varphi) \). The next lemma is proved in [2].

**Lemma 2.** Let \( L = \{ \varphi, \varphi_1 < \varphi < \varphi_2 \} \), where \( \lambda = \lambda_{n,k}(\varphi_1) = \lambda_{n,k}(\varphi_2) = \cot \alpha, \alpha = \alpha_{n,k}(\varphi_1) \) and \( \lambda_{n,k}(\varphi) \neq \lambda \) for \( \varphi \in L \), and assume that the point \( f_k(\varphi_1, \alpha) \) does not coincide with \( \varphi_1 \) for any \( j, 0 < j < 2n \), i.e., the corresponding cycle has precisely \( 2n \) links.

Then \( M_0, M_0 = \{ \varphi, \varphi_1 \varphi_2 \} \), exists such that the sets of points on
\[
M_n = \begin{cases} f_n(\varphi, \alpha, \varphi \in M_0 & n \geq 0, \\ f_n(\varphi, \alpha, -\alpha) \in M_0 & n \leq 0, \\ \end{cases}
M_\infty = \{ \varphi \in \Gamma, \theta_0 < \varphi < \psi_0 \}
\]
do not intersect, while
\[
\sum_{h=-\infty}^{\infty} M_{2n_k} = L.
\]

If the condition of the lemma is infringed, i.e., the points \( \varphi_1, \varphi_2 \) belong to the same cycle, having tangents as its two links, then the set \( M_0 \) can still be constructed by arguments similar to those used in the proof of Lemma 2.

Let \( R \) denote the set of all the \( \lambda \in [a, b] = [\min_{\varphi} \lambda_{n,k}(\varphi), \max_{\varphi} \lambda_{n,k}(\varphi)] \) for which the equations \( \lambda = \lambda(\varphi, \alpha) \), \( \lambda(\varphi) = 0 \) hold for a certain \( \varphi_0 \). We shall require the following, proved in [3]:

**Lemma 3.** Let \( \lambda \in (a, b), \lambda \notin R, h \in W_2^1 \cap C^2 \) and \( (\Delta h, \chi) \varphi = 0 \) for all the \( \chi \), representing the solution of the problem \( -\Delta^2(g^2/\partial y^2) + [\lambda^2/(1 + \lambda^2)]v = 0, v|_\Gamma = 0 \). The equation \( -\Delta^2(g^2/\partial y^2)u + [\lambda^2/(1 + \lambda^2)]u = h \) then has a solution \( u \in W_2^1(\Omega) \).

Let \( \Omega \) be a domain whose boundary satisfies the assumptions made at the start of the article. We shall henceforth assume that \( \lambda \notin R \). We denote by \( M \) the linear manifold of solutions of the problem
\[
-\lambda^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad u|_\Gamma = 0.
\]
Then \( L_2(\Omega) = M \oplus M^1 \), where \( M^1 \) is the orthogonal complement of \( M \) in \( L_2(\Omega) \), and every element \( h \) of \( L_2(\Omega) \) can be expressed uniquely as \( h_1 + h_2 \), where \( h_1 \in M, h_2 \in M^1 \). Notice that, if the element \( h \) is orthogonal to all solutions of the problem (6)-(7), then \( h_1 = 0, h_2 = h \).

Consider the problem
\[
-\lambda^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = g, \quad u|_\Gamma = 0.
\]
We shall construct a linear operator \( N^{-1} \), defined in the whole of \( L_2(\Omega) \), which associates with every element \( g \in L_2(\Omega) \) a generalized solution \( u = N^{-1}g \) of the problem (6')-(7'); notice that \( N^{-1}g \) belongs to \( \hat{W}_2^1(\Omega) \) in cases