using sequence (17) constructed for the function \( f(z) = e^{Hz} \) with respect to the node matrix \( \{ z_j^{(2n)} \} \) diverges for any \( z > 0 \) even though all the hypotheses of Theorem 1 are fulfilled when \( \varepsilon = 0 \).

LITERATURE CITED

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TIME OF FIRST ENTRY INTO A REGION WITH CURVED BOUNDARY

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1. Introduction

1. Preliminary Information. Let \( \xi_n \) be a sequence of independent identically distributed random variables. Consider the random walk \( (S_n) \) generated by the sequence of sums \( S_0 = 0, S_1 = \xi_1, S_n = \xi_1 + \ldots + \xi_n \). We will be interested in the time of first entry of the random walk into a certain set of real numbers which varies with time. More precisely, let \( \varphi = (\varphi_n) \) be a sequence of real numbers. In this paper we consider the random variable

\[ \eta = \eta_0 = \inf \{ k \geq 1 : S_k \geq \varphi_k \}. \]

This is the time of first entry of the random walk \( (S_n) \) into the set \( \{ \varphi_n, \infty \} \). The sequence \( \varphi = (\varphi_n) \) will be called a boundary.

This paper is devoted to studying the behavior of the sequence \( \mathbb{P}(\eta > n) \). The simplest of the questions which arise here is that of the existence of the mathematical expectation of \( \eta \). In addition, it is natural to study the question of the existence for \( \eta \) of moments of arbitrary order. Asymptotic estimates for the sequence \( \mathbb{P}(\eta > n) \) in terms of the jump distribution of the random walk \( \xi_i \) are among the more subtle problems.

We will not stop here to give a survey of the numerous papers devoted to the study of the distribution of \( \eta \) for a random walk generated not by a sequence of random variables, but by a sequence of series of random variables. A large number of results and a bibliography concerned with this class of questions may be found in [1] by Borovkov.

Results related to the existence of moments of \( \eta \) for a boundary \( \varphi \) of the form \( \varphi_n \sim -cn, c > 0 \), are contained in Gut [2]. In this paper we make an attempt to answer questions related to the behavior of the sequence \( \mathbb{P}(\eta > n) \) for a class of boundaries \( \varphi \) larger than in [2]. The main requirements which we impose on the
boundaries can be characterized by the words "approximate convexity," so that, in particular, domains of the form \( \varphi_n \sim \alpha n^2 \), \( c > 0 \), \( 0 < \alpha \leq 1 \) satisfy the requirements. The results of this paper were announced in [3].

The method employed in this paper consists in the following. If we consider "convex" boundaries \( \varphi \) (cf. Sec. 2 for a precise definition), then it turns out that the generating function of the sequence \( (P(\eta > n)) \) is an analytic function of the generating function of the sequence \( (n^{-1}P(\varepsilon_{\xi_k}^n)) \), where \( E_{\xi_k}^n \) is the event determined by cyclic permutation of the trajectories \( S_1, \ldots, S_n \) (cf. Sec. 2, Theorem 1, and [4]). Using Banach algebra methods, we can reduce the study of the sequence \( (P(\eta > n)) \) to the investigation of the sequence \( (n^{-1}P(\varepsilon_{\xi_k}^n)) \). Papers [1] of Borovkov, [5] of Essen, and [6-8] of Rogozin are devoted to the development of the Banach algebra method as it applies to such problems. We will use the Banach algebras introduced by Rogozin. Below we state a theorem which can be extracted from [6-8].

The behavior of the sequence \( (n^{-1}P(\varepsilon_{\xi_k}^n)) \) is determined by that of the sequences \( (n^{-1}P(S_n < \varphi_n)) \) and \( (P(\xi_1 < \varphi_n)) \). The Nagaev–Fuk inequality [9], a statement of which we also give here, is used to establish relations among these sequences.

2. Theorem of Rogozin. A sequence of positive numbers \( \tau = (\tau_n) \) is said to be hyperpower if

\[
K_\tau = \sup_{n > 1} \sup_{k=2} n \tau_k / \tau_n < \infty.
\]

A nondecreasing sequence \( \gamma = (\gamma_n) \) of positive numbers is called semimultiplicative if

\[
\gamma_n \gamma_m \geq \gamma_{n+m}, \quad \gamma_n \geq 1.
\]

For certain sequences (hyperpower \( \tau \) and semimultiplicative \( \gamma \)) we consider the set \( R_{\tau\gamma} \) of power series

\[
a(z) = \sum_{h=0}^{\infty} a_h z^h,
\]

and assume that

\[
P_{\tau\gamma}(a) = \sup_{h > 0} a_h |\tau_h + \sum_{h=0}^{\infty} \gamma_h a_h| < \infty.
\]

\( R_{\tau\gamma} \) is a Banach algebra with norm \( \|a\| = K_{\tau\gamma} P_{\tau\gamma}(a) \).

If the sequence \( \tau \) is such that \( \tau_n \equiv 1 \) and the sequence \( \gamma \) is defined by the equalities \( \gamma_n = t \ln n \), where \( t > 0 \), then the corresponding Banach algebra \( R_{\tau\gamma} \) will be denoted by the symbol \( R_t \).

If the sequence \( \gamma \) is such that \( \gamma_n \equiv 1 \), and \( \tau \) is arbitrary, the corresponding ring will be denoted by the symbol \( R_{\gamma} \).

THEOREM A [6-8]. Let \( x \in R_{\tau\gamma} \), \( A(y) \), a function analytic in a domain \( D \) containing the set \( \{ y : y = x(z), |z| < 1 \} \). Then the series \( (Ax(z)) = \sum_{h=0}^{\infty} A_h z^h \) also belongs to the algebra \( R_{\tau\gamma} \).

3. Nagaev–Fuk (N–F) Inequality. Let \( \mu(y) = M(\xi_1 | \xi_1 | < y), a(t, y) = M(t \xi_1 | \xi_1 | < y) \).

THEOREM B [9]. For \( 1 \leq t \leq 2 \) and \( x > 0, y > 0 \), the N–F inequality

\[
P(S_n < x) \leq n P(\xi_1 < y) + P(x, y)
\]

holds, where

\[
P(x, y) = \exp \left( \frac{x}{y} \left( \frac{x - n\mu(y)}{y} + \frac{n a(t, y)}{y^t} \right) \ln \left( \frac{y^{t-1}}{n a(t, y)} + 1 \right) \right).
\]

2. Exact Relations

1. A Combinatorial Lemma. In this section we assume that the sequence of numbers \( \varphi = (\varphi_n) \) forming the boundary possesses the following property: (\( \varphi_1 \))

\[
\varphi_n + \varphi_k \leq \varphi_{n+k}, \quad \text{for all positive integers } n, k.
\]

Given a vector \( a = (a_1, \ldots, a_n) \in \mathbb{R}^n \), we define a number by

\[
\eta_1(a) = \inf \{ k \geq 1 : a_k + \ldots + a_n \geq \varphi_n \},
\]

provided this minimum exists. The variable \( \eta_1 \) will be called the first ladder moment for the vector \( a \). If \( \eta_1 \) exists, we introduce another number

\[
\eta_2(a) = \inf \{ k \geq 1 : a_{n+k} + \ldots + a_{n+k} \geq \varphi_k, k \leq n - \eta_1 \}.
\]

The number \( \eta_1 + \eta_2 \) is called the second ladder moment of \( a \). There are at most \( n \) such ladder moments for \( a = (a_1, \ldots, a_n) \).