By virtue of (47) (cf. [7], Theorem 3.18), we can assert that the perturbed operator \( P \) (as well as the unperturbed one \( P_1 \)) will have in the domain \( |\lambda| > \lambda_1/2 \) a unique number belonging to the spectrum. Moreover, this will be a simple eigenvalue. By direct verification we find that this eigenvalue is equal to one, and the corresponding eigenfunction is \( \psi \equiv 1 \). Consequently, there exists a unique probability measure \( Q \), invariant with respect to \( P \), whose density belongs to \( L^p(\nu) \).

Now applying Theorem 1 of [4] and Theorem 5.5.6 of [9], we conclude that the transition operator \( P \) in \( L^p(\nu) \) satisfies (1).

On the other hand, it is obvious that the Markov chain investigated does not satisfy the condition of uniformly strong mixing.

**LITERATURE CITED**


**ASYMPTOTIC EXPANSIONS FOR DISTRIBUTIONS OF SUMS OF INDEPENDENT LATTICE RANDOM VECTORS**

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**UDC 519.21**

**Introduction**

Let \( \xi_j = (\xi_{j1}, \ldots, \xi_{jk}) \), \( j = 1, \ldots, n \), be independent, not identically distributed lattice random vectors of the Euclidean space \( \mathbb{R}^k \), assuming values from the common lattice

\[
\{(m_1, \ldots, m_k): m_i = 0, \pm 1, \pm 2, \ldots; i = 1, k\}.
\]

For simplicity of notation, let us assume that the expectations of the components of the vector \( \xi_j \), \( j = 1, n \), are equal to zero, \( V_j(x) \), \( v_j(t) \) are the respective distribution and characteristic functions of the random vector \( \xi_j \), \( j = 1, n \); \( t, x \) is the scalar product of the vectors \( t = (t_1, \ldots, t_k) \) and \( x = (x_1, \ldots, x_k) \) from \( \mathbb{R}^k \); \( |t| \) and \( |x| \) are the norms of the vectors \( t \) and \( x \) in \( \mathbb{R}^k \); \( U \) is the class of all Borel sets in \( \mathbb{R}^k \); \( \sigma_j^2 \) are the variances of the random variables \( j = 1, n, l = 1, k \);

\[
s_j = (\sigma_{j1}, \ldots, \sigma_{jk}), j = 1, n;
\]

\[
B_{jl} = \sum_{j=1}^{n} \sigma_{jl}^2, B_n(B_{l1}, \ldots, B_{lk});
\]

\[
\bar{\sigma}^2 = B_{ll}/n, \quad \bar{\sigma} = (\bar{\sigma}_1, \ldots, \bar{\sigma}_k);
\]

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\( \gamma_j = \frac{s_j}{a} = \left( \frac{s_{j1}}{a_1}, \ldots, \frac{s_{jk}}{a_k} \right) \) is a random vector from \( \mathbb{R}^k \) with distribution function \( F_j(x) \) and characteristic function \( f_j(t) \); \( \Theta = (\theta_1, \ldots, \theta_k) \) is a random vector from \( \mathbb{R}^k \) with distribution function

\[
F(x) = \frac{1}{n} \sum_{j=1}^{n} F_j(x);
\]

\( V \) is the correlation matrix of the vector \( \Theta \); \( Q(t) = M[\Theta, t] \) is a quadratic form; \( \beta_r(t), \chi_r(t) \) are respectively the absolute moment and semi-invariant of the \( r \)-th order of the random variables \( \Theta, t \); \( \beta_{rj}(t), \chi_{rj}(t) \) are respectively the absolute moment and semi-invariant of the \( r \)-th order of the random variable \( \gamma_j(t), j = 1, n \); \( F_{N_r}(x) \) and \( F_{N_r}(t) \) are respectively the distribution function and characteristic function of \( S_n \), where

\[
S_n = \sum_{j=1}^{n} \left( \frac{s_{j1}}{\sqrt{a_1}}, \ldots, \frac{s_{jk}}{\sqrt{a_k}} \right) = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} \Theta_j;
\]

\( \beta_r = M[(\Theta V^{-1} \Theta)^r]; \beta_{rj} = M[\gamma_j V^{-1} \gamma_j]^r; \)

\( p_i = \left\{ \xi_j = i \right\}, \xi_j = (\xi_{j1}, \ldots, \xi_{jk}), j = 1, n; \)

\[ m = \left\{ (m_1, \ldots, m_k) ; m_i = 0, \pm 1, \pm 2, \ldots; i = 1, k \right\}; \]

\( P_n(A) = \left\{ S_n \in A \right\}, A \subset U. \)

We shall need the polynomials \( P_r(\omega) \), which are defined from the following formal expansions:

\[
\exp \left\{ \sum_{r=1}^{\infty} \frac{r^2 + (\omega)}{(r+2)!} \omega^r \right\} = 1 + \sum_{r=1}^{\infty} P_r(\omega) \omega^r;
\]

\( P_j(\Theta, -\phi)(x) = \int_{x_k}^{\infty} \ldots \int_{x_1}^{\infty} P_j(\Theta, -\phi)(y) dy, \)

\( P_j(\Theta, -\phi)(x) = \frac{1}{J^k} \int_{J_1}^{J_k} P_j(\Theta, -\phi)(x) dx, \)

\[
\lim_{n \to \infty} \inf \lambda_n > 0; \quad \lim_{n \to \infty} \frac{B_{ij}}{n} > 0, \quad \lim_{n \to \infty} \sup \frac{1}{n} \sum_{j=1}^{n} M(\xi_j)^r < \infty;
\]

\[
\frac{1}{n} \sum_{j=1}^{n} \sum_{j=1}^{n} m_i p_{ij} \to 0 \quad \text{for} \quad n \to \infty,
\]

where \( \varphi(x) \) and \( \Phi(x) \) are respectively the density and distribution function of the normal law with parameters \( (0, V) \).

In the present paper we estimate the remainder term of the asymptotic expansion for the distribution function of a sum of independent nonidentically distributed lattice multidimensional random variables with values on the common lattice of integral vectors.

For \( k = 1 \) this question was studied profoundly by Mitalauskas and Statulyavichus [6].

For \( k > 1 \) for the case of identically distributed summands, Bikyalis in [4] obtained necessary and sufficient conditions.

1. Formulation of the Results

We write conditions which we shall need later.

Let us assume that \( s > 3 \), and let \( \lambda_n \) be the least eigenvalue of the correlation matrix \( V \).

Without loss of generality, let us assume that

\[ p_j / p_{jm} \]

for all \( j = 1, n \) and all \( m \) from (1).

Suppose further

\[
\lim_{n \to \infty} \lambda_n > 0; \quad \lim_{n \to \infty} \frac{B_{ij}}{n} > 0, \quad \lim_{n \to \infty} \sup \frac{1}{n} \sum_{j=1}^{n} M(\xi_j)^r < \infty;
\]

\[
\frac{1}{n} \sum_{j=1}^{n} \sum_{j=1}^{n} m_i p_{ij} \to 0 \quad \text{for} \quad n \to \infty,
\]

for all \( j = 1, n \) and all \( m \) from (1).