The last equation is true due to the corollary to Lemma 3, when the number p is substituted for t. The proof of the lemma is finished.

To prove the fact that \( \mathbb{P}(\mathbb{Z}) \) is everywhere dense in \( \mathbb{R}(\mathbb{Z}) \), we note that the characteristic function of a shifted open set \( x_0 + U_N, x_0 \in \mathbb{Z}_p \) is obtained from the characteristic function \( \chi_N \) by shifting the argument by the p-adic integer \( x_0 \), i.e., \( \chi_{N+x_0}(x) = \chi_N(x+x_0) \). The linear subspace generated by the characteristic functions \( \chi_N \) with arguments shifted by p-adic integers over the field of rational numbers is everywhere dense in the ring \( \mathbb{R}(\mathbb{Z}) \).

From this remark and the theorem it follows that the ring \( \mathbb{P}(\mathbb{Z}) \) is everywhere dense in \( \mathbb{R}(\mathbb{Z}) \).

LITERATURE CITED

ACCURACY OF APPROXIMATION OF SUMS OF INDEPENDENT RANDOM VARIABLES BY A STABLE DISTRIBUTION

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1. Introduction

We consider a sequence of independent random variables (r.v.) \( \xi_1, \xi_2, \ldots, \xi_n \) with common distribution function (d.f.) \( F(x) \) and characteristic function (c.f.) \( f(t) \). Let \( G_\alpha(x) \) and \( g_\alpha(t) \) be the d.f. and c.f. of a stable law with exponent \( \alpha \), \( 0 < \alpha \leq 2 \). Without loss of generality, let us assume that

\[
\log g_\alpha(t) = \begin{cases} 
-|t|^\beta \operatorname{sgn} t \tan \left( \frac{\pi \alpha}{2} \right), & \text{if } 0 < \alpha < 2, \alpha \neq 1, \\
-|t| \left( 1 + i (2/\pi) \beta \operatorname{sgn} \ln |t| \right), & \text{if } \alpha = 1, \\
-|t|^2/2, & \text{if } \alpha = 2,
\end{cases}
\]

where \( \beta \in [-1, 1] \).

We suppose that the d.f. \( F(x) \) belongs to the zone of normal attraction of the law \( G_\alpha(x) \). In the present paper we shall consider the behavior of the distribution of the sum

\[ Z_n = n^{-\tau} \left( \sum_{j=1}^{n} \xi_j - A_\alpha \right), \]

where \( \tau = 1/\alpha \),

\[ A_\alpha = \begin{cases} 
0, & \text{if } \alpha < 1, \\
2\beta n \ln n/\pi, & \text{if } \alpha = 1, \\
n \beta n^{1/2}, & \text{if } \alpha > 1.
\end{cases} \]
Without loss of generality, let us assume that $E_1 = 0$, if $a > 1$, and $D_1 = 1$, if $a = 2$.

We write $\Delta(x) = F(x) - G_\alpha(x)$, $\Delta_0(x) \equiv 0$, $\Delta_1(x) \equiv \Delta(x)$,

$$\Delta_k(x) = \begin{cases} -\int_x^\infty \Delta_{k-1}(y) dy, & x \geq 0, \\ \int_x^{-\infty} \Delta_{k-1}(y) dy, & x < 0, k \geq 2, \end{cases}$$

and let $F_n(x)$ and $f_n(t)$ be the d.f. and c.f. of the sum $Z_n$, $\Delta_{ma}(x) = F_n(x) - G_\alpha(x)$, $\Delta_{ma} = \sup \{ \Delta_{ma}(x) \}$.

Many authors have been concerned with estimates of the rate of convergence of the error term $\Delta_{ma}$. In many papers for rate of convergence of a specific order one requires the existence of either absolute pseudomoments (cf. [1, 12-14], etc.), or of absolute difference moments (cf. [13, 15, 16], etc.), or of truncated pseudomoments (cf. [3, 17], etc.) of corresponding order. But at the same time there emerge examples (cf. [15, Proposition 1]) showing that a good rate of convergence can also be obtained in the case when the above-mentioned characteristics of the corresponding order are infinite.

To improve the order of the rate of convergence of $\Delta_{ma}$, Christoph [11] introduced the integral difference moments

$$\gamma_r = \int_{-\infty}^\infty |x|^{-[r]} |d\Delta_{[r]+1}(x)|,$$

and in [21] the truncated integral difference moments

$$\gamma_r = \left\{ \begin{array}{ll} \sup_{z > 0} z^{-[r]} \int_{|x| > z} |d\Delta_{[r]+1}(x)|, & r \neq [r], \\ \sup_{z > 0} \left\{ \left| \int_{|x| > z} \Delta_r(x) dx \right| + \int_{|x| < z} |d\Delta_r(x)| \right\}, & r = 1, 2, \ldots. \end{array} \right.$$  \hspace{1cm} (2)

In [11] he showed that for $v_r < \infty$, $r \leq 1 + \alpha$ if

$$(D_0) \int_{-\infty}^\infty \Delta_i(x) dx = 0, \quad i = 0, [r]$$

then $\Delta_{ma} = O(n^{-[r]-\alpha})$, $n \to \infty$; and in [21] he obtained the rate of convergence of $\Delta_{ma}$ of the same order for $\gamma_r < \infty$, $r \leq 1 + \alpha$ and (2). The question arises does one have rate of convergence of the above-mentioned order for $\gamma_r = \infty$ and $v_r = \infty$.

In connection with this, we consider the following example. Let $1/3 < \alpha < 1/2$, the r.v. $X_i^{(a)}$, $i = 1, 2, \ldots$, have density

$$p_{\alpha}(x) = \Gamma(\alpha + 1) \sin(\pi \alpha/2)/|x|^{1+\alpha}.$$

It is easy to show that $\mathbb{E} \{ t X_{[t]}^{(a)} \} = (|t+1|^\alpha + |t-1|^\alpha)/2 - |t|^\alpha$ and the distribution of the r.v. $X_{[t]}^{(a)}$ belongs to the domain of normal attraction of the law $G_\alpha(x)$ with $\beta = 0$. We note that $|\mathbb{E} \{ t X_{[t]}^{(a)} \} - \exp(-|t|^\beta)| = O(|t|^{2\alpha})$ as $|t| \to 0$ and according to the result of V. Paulauskas [15, Proposition 1],

$$|P\{X_{[t]}^{(a)} + \ldots + X_{[t]}^{(a)}/n^\beta < x\} - G_\alpha(x)| = O(n^{-1}), \quad n \to \infty.$$  \hspace{1cm} (4)