This paper is in many respects a continuation of [1]. A brief account appeared in the note [2].

As we showed in [1], many aspects of strongly minimal countably categorical structures or, equivalently, countable minimal structures definable in models of theories which are categorical in any infinite cardinal (in totally categorical structures) are characterized by the properties of the combinatorial geometries connected with these structures. In this paper this connection is made more precise and a complete characterization of the aforementioned geometries is offered.

The paper is addressed to readers who are familiar with the techniques of contemporary stability theory. However, its main result should be accessible to readers whose specialization is far from model theory and, in our opinion, it should interest specialists in combinatorial geometries.

We start by reviewing the basic notions used in the paper. A set D equipped with a closure operator c acting on the family of all its subsets is called a pregeometry [3] if

\[ X \subseteq c(X); \]
\[ c(X) = \bigcup \{c(X') : X' \subseteq X, X' \text{ is finite}\}; \]
\[ X \subseteq c(Y) \implies c(X) \subseteq c(Y); \]
\[ \text{for any } x, y \in D, x \in c(X \cup \{y\}) \implies y \in c(X \cup \{x\}). \]

A pregeometry is called a geometry if

\[ c(\{x\}) = \{x\} \text{ for all } x \in D. \]

A pregeometry is said to be locally finite if \( c(X) \) is finite for any finite \( X \).

With each pregeometry \( D \) one can associate naturally a geometry \( \hat{D} \): the points of \( \hat{D} \) are the sets \( c(\{x\}) \) with \( x \in D \setminus c(\emptyset) \), and the closure of the subset

\[ X = \{c(\{x\}) : x \in X \setminus c(\emptyset)\} \subseteq \hat{D} \]

is

\[ c(X) = \{c(\{y\}) : y \in c(X \setminus c(\emptyset))\}. \]

A pregeometry \( D \) is said to be homogeneous if for any \( X \subseteq D \) and any points \( x, y \in D \setminus c(X) \) there is a pregeometry automorphism (i.e., a mapping preserving \( c \)) which takes \( x \) into \( y \) and is identical on \( X \).

As it was shown in [4], the definition of the notion of algebraic closure \( c^p \) proposed by Marsh satisfies all conditions (1)-(4) on any strongly minimal set \( D \). Therefore, on any strongly minimal structure there is defined a pregeometry, which will be denoted by the same letter. Moreover, it is known that for a strongly minimal \( D \) the pregeometry is homogeneous, and if the theory \( D \) is countably categorical then it is also locally finite.

The proof of the following proposition which, in a certain sense, is a converse of the previous assertion is an easy exercise.

Let \( <D, c> \) be an infinite, locally finite pregeometry. Consider the structure \( <D, \{c(n) : n = 1, 2, \ldots\}> \), where \( c(n)(x_1, \ldots, x_{n-1}, y) \) is the \( n \)-place predicate on \( D \) meaning that \( \forall \in c(\{x_1, \ldots, x_{n-1}\}) \). Then this structure is countably categorical and strongly minimal, and the operators \( c \) and \( c^p \) coincide.
The last observation permits us to regard infinite, locally finite, homogeneous pre-geometries as countably categorical strongly minimal structures.

Let V be a vector space over the field F. If for arbitrary $X \subseteq V$ we set $c(X)$ equal to the smallest vector subspace of $V$ containing $X$ we obtain a homogeneous pregeometry. A geometry isomorphic to $V$ for some vector space $V$ will be referred to as a projective geometry over the field $F$.

There is another geometry connected with a given vector space $V$. Indeed, take for $c(X)$ the smallest subset of $V$ which contains, along with any pair of distinct points $v_1, v_2$, all points $v_1 + a(v_1 - v_2)$ with $a \in F$ (that is, the line passing through $v_1$ and $v_2$). Any geometry isomorphic to the one we just indicated is called an affine geometry over the field $F$ [3]. Projective and affine geometries over a field are homogeneous; if the field is finite they are also locally finite.

It is well known [3] that the affine and projective geometries of dimension greater than two can be characterized in terms of relations between subspaces and hence also in terms of the closure operator. In particular, a geometry $D$ is projective if and only if

\[ \text{for any } y \in D, Z \subseteq D, x \in c(\{y\} \cup Z) \]
\[ \text{one can find } z \in c(Z) \text{ such that } x \in c(\{y, z\}), \]

and

\[ \text{for distinct } x, y \in D \mid |c(\{x, y\})| \geq 3. \]

We say that pregeometry $D$ is degenerate if in geometry $D$ the equality $c(X) = X$ holds for all $X \subseteq D$.

A strongly minimal structure whose geometry is degenerate will be referred to as disintegrating [1].

**Main Theorem.** Given any strongly minimal countably categorical disintegrating structure the corresponding geometry is either projective or affine over a finite field.

As we already remarked, the proof of this (essentially combinatorial-geometric) theorem uses the techniques of contemporary theory of stability. We are however convinced that in the present case these model-theoretic concepts can be, in principle, reduced to natural combinatorial-geometric concepts. This would provide a proof which is independent of model theory and has essentially an elementary character. We also hope that in this manner it would be possible to formulate and prove a considerably stronger theorem and, in particular, weaken the infiniteness and local finiteness requirements. We further note that after becoming acquainted with the author's work [1], Cherlin observed that the main theorem can be obtained as a corollary of the classification of all finite doubly transitive groups. The proof of this classification theorem rests on the classification of all finite simple groups. At the present time the latter is considered to be complete, although, to the best of our knowledge, an intuitive proof is not yet available.

We note that the main theorem describes also the infinite, locally finite, homogeneous geometries. In fact, the isomorphism type of pregeometry $D$ is completely specified by the isomorphism type of the geometry $D$ and the number of elements in $c(\emptyset)$ and $c(\{x\})$ for an arbitrary $x \in D\setminus c(\emptyset)$.

Unfortunately, in the general case the description of pregeometries of minimal countably categorical structures $D$ does not yield a description of all possible definable predicates on $D$, although it could serve as a good basis for solving this important problem (Theorem 6.7).

Notice that from the main theorem it follows, according to [1, 5], that there is no complete finite axiomatizable theory which is categorical in all infinite cardinals. A first proof of this fact was proposed by the author in [6, 7].

The main theorem is also the starting point of paper [8], to be published soon. In this reference the theorem asserting the impossibility of a finite axiomatization is extended to include complete, countably categorical, totally transcendental theories; therein it is also proved that the Morley rank of such categories is finite.

Let us give a clear outline of the initial data of the proof of the main theorem.

Suppose the signature of the structure $D$ is extended by the symbols of constants for all elements of some set $A$. Then the operator $c_L$ changes, and we will use $D_A$ to denote the new