§ 1. Introduction

We consider a system

\[
\begin{align*}
\frac{d\xi}{dt} &= \xi, \\
\frac{d\psi}{dt} &= -\varphi(\xi, \sigma) - x^*Cf(\sigma) + \varphi(\sigma), \\
\frac{dx}{dt} &= Ax + Bf(\sigma)\xi,
\end{align*}
\]

where A is a constant Hurwitz (n × n) matrix, B and C are constant (n × m) matrices, f(\sigma) is a \(\Delta\)-periodic differentiable vector-function, \(\varphi(\sigma)\) is a continuously differentiable \(\Delta\)-periodic function, and \(\varphi(\xi, \sigma)\) is a differentiable function satisfying the following relations for all \(\sigma\) and \(\xi\)

\[
\mu_1\xi^2 \leq \varphi(\xi, \sigma) \xi \leq \mu_2\xi^2, \quad \varphi(\xi, \sigma+\Delta) = \varphi(\xi, \sigma).
\]

Here \(\mu_1\) and \(\mu_2\) are certain numbers.

Problems occurring in the analysis of synchronous motors [1-3], as well as problems involving Boisse-Sard regulators [4], lead to the systems of form (1).

In this paper we consider the question of stability "in the large" for system (1). The concept of stability "in the large" for such systems differs slightly from the traditional concept. This stems from the fact that the stationary set for system (1) is always either empty or infinite. Therefore, instead of the traditional concept of stability "in the large" we shall use the analogous concept of global asymptotic stability. In the theory of synchronous motions this latter concept is adequate since for any initial perturbation we have either synchronous dynamic or resultant stability.

In this paper we also give conditions for the existence of circular motions, and for the existence of limit cycles of second type, for system (1). In the case when system (1) describes the dynamics of a synchronous motion, such solutions correspond to a regime of operation for which the rotor of the synchronous machine makes rotations of an arbitrarily large electrical phase.

In this paper the investigation of system (1) will be reduced, by means of theorems to be presented below, to the investigation of the often-studied second-order equation

\[
\ddot{\sigma} + \alpha \dot{\sigma} - \varphi(\sigma) = 0,
\]

where \(\alpha\) is some positive number. By means of such a reduction we can succeed in extending to systems of form (1) well-known results of Tricomi [5], Amerlo [6], Seifert [7], Böhm [8], Hayes [9], Belyustina [10], and Tabueva [11], obtained for Eq. (3). Our methods of proof for the theorems given below differ substantially from the methods of these authors. In fact, we modify the second method of Lyapunov in such a way that it can be applied to the study of stability "in the large," as well as to the study of problems involving the existence of circular motions and limit cycles of the second type for systems with cylindrical phase spaces. To this end, conditions for the existence of appropriate functions of Lyapunov type are obtained by means of the "frequency theorem" of Yakubovich.

We remark that along with system (1) we shall often consider the system
\begin{align*}
\dot{s} &= \xi, \\
\dot{\xi} &= -\Psi(s) + y^* C \frac{d T}{d s} + \Psi(s), \\
\dot{y} &= Ay + B f(s) + q,
\end{align*}
which can be reduced to a system of form (1) by the so-called Lienard change of variables
\[y = x - A^{-1} \dot{B} f(s) - A^{-1} q.\]
Here
\[
B = A^{-1} \dot{B}, \quad f(s) = \frac{d T}{d s}, \\
\psi(s) = \frac{d T}{d s} \left[ C^* A^{-1} \dot{B} f(s) + C^* A^{-1} q \right].
\]
We present precise definitions.

Definition 1 [12-14]. System (1) is said to be globally asymptotically stable if every solution of this system converges to some equilibrium point as \(t \to +\infty\).

Definition 2 [14]. A solution \(X(t, X_0)\) of system (1) will be called circular if there exist numbers \(t_0\) and \(\varepsilon > 0\), such that for all \(t \geq t_0\) the following relation holds
\[
\| x(t, X_0) \| = \| \dot{x}(t, X_0) \| > \varepsilon.
\]
If system (1) is the system of equations for a perturbation of a synchronous motion, and if \(X(t, X_0)\) is a circular motion, then one can say [3, 15] that under "perturbation" \(X_0\) fails out of synchronous motion.

Definition 3 [14]. We shall say that a solution \(X(t, X_0)\) of system (1) is a limit cycle of second type, if there exists a number \(\tau > 0\) such that
\[
\begin{align*}
\sigma(t, X_0) - \sigma_0 &= \Delta, \\
\xi(t, X_0) - \xi_0 &= 0, \\
x(t, X_0) - x_0 &= 0.
\end{align*}
\]
We remark that while regimes of operation of synchronous motions corresponding to solutions \(X(t, X_0)\) of system (1) with the property
\[
\lim_{t \to +\infty} \sigma(t, X_0) = \infty,
\]
are undesirable, a Boisse-Sard regulator may operate only in a regime corresponding to solutions \(X(t, X_0)\) with property (5).

§ 2. Formulation of the Basic Results

We consider the \((m \times m)\) matrix \(U(p) = C^* (A - pI)^{-1} B\). Here \(p\) is a complex number.

**Theorem 1.** Let \(\mu_1 \geq 0\), \(\text{rang} B = m\), and suppose that the stationary set \(\Lambda\) of system (1) consists of isolated points and satisfies the following conditions:
1) for all \(\omega \in (-\infty, +\infty)\)
\[
U(i\omega) + U(i\omega)^* > 0,
\]
2) \(\lim_{\omega \to +\infty} \omega^2 (U(i\omega) + U(i\omega)^*) > 0\),
3) \(\int_{-\pi}^{\pi} \psi(s) ds = 0\),
4) for any number \(\sigma_0\) there exists a positive number \(\delta\), such that for all \(\sigma \in [\sigma_0 - \delta, \sigma_0 + \delta]\) and \(\sigma \neq \sigma_0\)
\[
B \int_{\sigma_0}^{\sigma} f(s) ds \neq 0.
\]