We consider the nonlinear ordinary differential equation

\[ L_{\varepsilon} = \sum_{k=0}^{n} (-1)^k \left[ p_k^\varepsilon (\bar{u}, t) \right]^{(k)} = 0, \]

where we have denoted: \( \bar{u} = (u^{(n)}, \ldots, u) \), with the boundary conditions

\[ u^{(l)}(a) = A_l, \quad u^{(l)}(b) = B_l, \quad l = 0, \ldots, n-1; \quad |a|, |b| < \infty. \]

The functions which occur in (1) depend on the parameter \( \varepsilon \in \mathbb{R} \) (e.g., periodic functions with period \( \varepsilon \)). The purpose of this paper consists in the construction of an equation, which does not contain the parameter \( \varepsilon \) and whose solution is the limit as \( \varepsilon \to 0 \) of the sequence of solutions of the problems (1), (2). We do not assume that the operator in (1) is defined for \( \varepsilon = 0 \) (this happens, e.g., when the functions in (1) depend on the argument \( t/\varepsilon \)). The case of the linear equation (1) has been considered, in particular, in [1, 2].

We impose on the functions \( \{p_k^\varepsilon\} \) the following conditions:

1. \( p_k^\varepsilon \in C^0(\mathbb{R}^{n+1} \times [a, b]), \) and, in addition, the functions \( (p_k^\varepsilon)^{-1}, p_k^\varepsilon \) are Lipschitzian with respect to the first \( n + 1 \) arguments with constants that do not depend on \( \varepsilon; f^{-1}(x, \bar{y}, t) \) denotes the inverse of the function \( f(x, \bar{y}, t) \) with respect to the first argument: \( f^{-1}(x, \bar{y}, t) = z \) if

\[ f(z, \bar{y}, t) = x \vee \bar{y} \in \mathbb{R}^n \vee t \in [a, b]. \]

2. \( \forall x_1 > x_2 \vee \bar{y}_1, \bar{y}_2 \in \mathbb{R}^{n+1} \)

\[ M (x_1 - x_2) \geq M^{-1} (p_1^\varepsilon (x_1, \ldots) - p_1^\varepsilon (x_2, \ldots)) \geq p (x_1 - x_2), \]

\[ M \parallel \bar{y}_1 - \bar{y}_2 \parallel_\mathbb{R}^{n+1} = \sum_{k=0}^{n} \left[ p_k^\varepsilon (\bar{y}_1, t) - p_k^\varepsilon (\bar{y}_2, t) \right] (\bar{y}_1 - \bar{y}_2)_h \geq m \parallel \bar{y}_1 - \bar{y}_2 \parallel_\mathbb{R}^{n+1}. \]

**Theorem.** If for \( \varepsilon \to 0 \) there exist the following limits in the weak topology \( \sigma(L_1([a, b]), L_\infty([a, b])) \) [4]:

\[ (p_k^\varepsilon)^{-1} (\bar{x}(t), t) \to p_k^{-1} (x(t), t) \quad \forall x(t) \in (L_2([a, b]))^{n+1}, \]

\[ (p_k^\varepsilon)^{-1} (p_n (\bar{x}(t), t), x(t), t) \to p_n (\bar{x}(t), t); \quad k = 0, \ldots, n-1, \]

where also the functions \( \{p_k\} \) satisfy the conditions 1, 2, then

\[ u - u_n \to 0 \text{ in } C^{n-1}([a, b]), \]

\[ u^{(n)} - (p_n^\varepsilon)^{-1} (p_n (\bar{u}_n(t), t), \bar{u}(t), t) \to 0 \text{ in } C([a, b]). \]

Here \( u_n(t) \) is the solution of the problem obtained from (1), (2) by changing the functions \( \{p_k^\varepsilon\} \) to \( \{p_k\} \); we have denoted \( \bar{u} = (u^{(n-1)}, \ldots, u). \)

Proof. We rewrite (1) in the form
\[ Lu = \sum_{k=0}^{n} (-1)^k u^{(2k)} = \sum_{k=0}^{n} (-1)^k \left[ \frac{P_k^c(\vec{u}, t)}{V} \right]^{(2k)}, \]  
\[ P_k^c(\vec{x}, t) = \frac{M_{k(t)} - \rho^c_k(\vec{x}, t)}{M}. \]  

We introduce the influence functions \( \{U_k(t, \tau), k = 0, \ldots, n\} \) as the solutions of the equations
\[ LU_k(t, \tau) = (-1)^k \delta^{(k)}(t - \tau) \]
with the homogeneous conditions (2). After this, the problem (1), (2) reduces to the following integrodifferential equation:
\[ u(t) = \sum_{k=0}^{n} \int_a^b U_k(t, \tau) P_k^c(\vec{u}, \tau) d\tau + u_0(t). \]  

where \( u_0(t) \) is the solution of the problem (3), (2) for \( P_k^c \equiv 0 \). We also introduce the function \( u_\alpha(t) \) as the solution of the equation
\[ \sum_{k=0}^{n} (-1)^k \left[ p_k(\vec{u}_\alpha, t) \right]^{(2k)} = 0 \]
with the boundary conditions (2), and we introduce the function \( u_1(t) \) by the formula
\[ p_n(\vec{u}_1, t) = p_n(\vec{u}_\alpha, t). \]

For the function \( u_1(t) \) we write the following representation:
\[ u_1(t) = \int_a^b N(t - \tau) u_\alpha^{(n)}(\tau) d\tau + l(t). \]

Here the function \( N(t) \) is determined by the condition \( N^{(n)}(t) = \delta(t) \), while \( l(t) \) is an arbitrary polynomial of degree at most \( n - 1 \), which will be chosen later. From the problem (6), (2) we have the relation between \( u_\alpha(t) \) and \( u_0(t) \):
\[ u_\alpha(t) = \sum_{k=0}^{n} \int_a^b U_k(t, \tau) P_k(\vec{u}_\alpha, \tau) d\tau + u_0(t), \]
where the functions \( \{P_k\} \) are defined similarly to \( \{P_k^c\} \) in (4).

We form the equation for the function \( v = u - u_1 \). Subtracting from Eq. (5) the representation (8) of the function \( u_\alpha \) and making use of (9) for the elimination of \( u_0 \), after non-fundamental but cumbersome computations we obtain
\[ v(t) = \sum_{k=0}^{n} \int_a^b U_k [P_k^c(\vec{u}, t) - P_k^c(\vec{u}_1, t)] d\tau + \int_a^b \left[ \int_a^b U_n \left[ u_\alpha^{(n)} - u_\alpha^{(n)} \right] d\tau + \int_a^b \left[ \Phi(\vec{u}_1, t) - \Phi(\vec{u}_\alpha, t) \right] d\tau \right]. \]  

We represent the function \( u_\alpha(t) \) in the form
\[ u_\alpha(t) = \int_a^b N(t - \tau) u_\alpha^{(n)}(\tau) d\tau + l(t). \]

Making equal the polynomials \( l(t) \) and \( n(t) \) and taking into account (7), we obtain that the free term in (10) is equal to
\[ \int_a^b \left[ \int_a^b U_n \left[ u_\alpha^{(n)} - u_\alpha^{(n)} \right] d\tau + \sum_{k=0}^{n-1} \int_a^b \left[ \int \Phi(\vec{u}_1^{(n)} - \vec{u}_\alpha^{(n)}) d\tau \right] d\tau + \int_a^b \left[ \int \Phi(\vec{u}_1^{(n)} - \vec{u}_\alpha^{(n)}) d\tau \right] d\tau \].  

368