When various methods of measuring frequency deviation of frequency-modulated sinusoidal signals were studied at the Kharkov State Institute of Measures and Measuring Instruments, the oscillographic method was included. This method is a version of the instantaneous frequency-measuring method described in [1, 2] and differs from the latter in using an asymmetrical oscillogram peculiar to the extremal values of the frequency versus time function instead of the symmetrical one, obtained near the bend of the frequency versus time curve.

The frequency-modulated voltage \( u = U_m \sin(\omega t + \frac{\Delta \omega}{\Omega} \sin \Omega t + \alpha) \) is mixed with the "search" voltage \( u = U_m \cos \omega_0 t \) (here \( \omega/2\pi \) is the mean frequency of the frequency-modulated voltage; \( \Omega/2\pi \) is the modulating frequency; \( \alpha \) is an arbitrary phase, and \( \omega_0/2\pi \) is the "search" frequency).

The mixing of these frequencies produces a difference-frequency component

\[
 u(t) = U_m \sin \left( (\omega - \omega_0) t + \frac{\Delta \omega}{\Omega} \sin \Omega t + \alpha \right).
\]  

The "search" frequency may be outside or inside the swing of the frequency-modulated voltage.

Let us examine in detail the border case when \( (\omega - \omega_0) = \Delta \omega \); and \( \cos \Omega t = 1 \), i.e., when \( t = \frac{\pi}{\Omega} \). The instantaneous frequency at that point will equal zero, and the voltage \( U(t) \) will remain constant in its vicinity. Let us now examine at that point the function represented by (1). The voltage has the form

\[
 U \left( \frac{\pi}{\Omega} \right) = U_m \sin \left( \pi \frac{\pi}{\Omega} + \alpha \right)
\]  

or

\[
 U \left( \frac{\pi}{\Omega} \right) = U_m \sin \left( \frac{\Delta \omega}{\Omega} \pi + \alpha \right)
\]

For

\[
 0 < \frac{\Delta \omega}{\Omega} \pi + \alpha < \pi \text{ we get } U \left( \frac{\pi}{\Omega} \right) > 0;
\]

and for

\[
 \pi < \frac{\Delta \omega}{\Omega} \pi + \alpha < 2\pi \text{ we get } U \left( \frac{\pi}{\Omega} \right) < 0.
\]

The derivative of function \( U(t) \) is

\[
 \frac{d U}{d t} = U_m \cos \left( (\omega - \omega_0) t + \frac{\Delta \omega}{\Omega} \sin \Omega t + \alpha \right) \left[ (u - \omega_0) + \Delta \omega \cos \Omega t \right].
\]
Then,
\[ \frac{dU}{dt} \left( \frac{\pi}{\Omega} \right) = 0, \]

i.e., at that point the curve is parallel to the \( T \) axis and there is a characteristic horizontal stretch of the curve near that point.

It can be shown that
\[ \frac{d^2u}{dt^2} \left( \frac{\pi}{\Omega} \right) = 0 \]

and that
\[ \frac{d^2u}{dt^2} \left( \frac{\pi}{\Omega} \right) = U_m \cos \left( \frac{\Delta \omega \pi}{\Omega} \right) \Delta \omega \Omega \neq 0. \]

Thus at \( t = \pi / \Omega \) the function has a point of inflection.

Also \( \frac{du}{dt} = 0 \) for \( \left( \frac{\Delta \omega}{\Omega} \pi + \alpha \right) = \pi / 2 \) and for \( t = \pi / \Omega \), i.e., for the value of the phase when \( u = U_m \). For these values there exists a maximum of a higher order.

In examining the voltage \( u(t) \) to the left and the right of \( t = \pi / \Omega \), i.e., for \( t = \pi / \Omega + \tau \), it is possible to show that for the voltage is larger for \( \tau < 0 \) than for \( \tau < 0 \).

For \( \left( \frac{\pi \Delta \omega}{\Omega} + \alpha \right) > \frac{\pi}{2} \) the reverse relation holds. The family of curves in the vicinity of point \( t = \pi / \Omega \) is shown in Fig. 1.

Thus the characteristic property of the function under consideration for \( t = \pi / \Omega \) and \( \omega - \omega_0 = \Delta \omega \) consists in horizontal stretches of the function curve situated at different levels according to the value of \( \alpha \).

If the voltage represented in (1) is fed to the vertical deflecting plates of the oscilloscope and modulating voltage \( U_\omega \cos \Omega t \) to the horizontal plates the following pattern will be observed:

a) when \( \omega_0 > \omega + \Delta \omega \), i.e., when \( \omega_0 \) is outside the frequency swing. In this case a characteristic frequency modulated curve is observed. Since function \( \sin \left( (\omega - \omega_0) t + \frac{\Delta \omega}{\Omega} \sin \Omega t + \alpha \right) \) is not periodic, the scanning by means of frequency \( F = \Omega / 2\pi \) produces a moving curve (Fig. 2a).

b) when \( \omega_0 \) is near to \( \omega + \Delta \omega \). A bend appears at point \( t = \pi / \Omega \) (Fig. 2b).

c) when \( \omega_0 - \omega = \Delta \omega \). In this case Fig. 2b changes to Fig. 2c with its characteristic horizontal stretch in the vicinity of point \( t = \pi / \Omega \), a stretch whose distance from the horizontal axis is equal to \( U_m \sin(\pi \Delta \omega / \Omega + \alpha) \), i.e., depends on the value of \( \alpha \).