ESTIMATION OF POTENTIALITIES OF METHODS OF SUMMATION OF DIRECT MEASUREMENT ERRORS

É. G. Mironov and G. Zh. Orduyants

Conventional methods for summation of direct measurement errors are considered. Conditions under which these methods provide consistent results are stated. It is proposed to utilize for characteristic of the total error, its interval estimator or the half-sum of the value obtained by adding up its arithmetic and geometric random errors.

As is well known, a rigorous summation of errors of direct measurements requires consideration of numerous factors and a large amount of additional information concerning the quantities to be summed up [1-3]. These factors are statistical distribution laws of the individual summands, the presence or absence of correlations, their dependence on the quantities subject to measurements, and so on.

Usually this information is not available, its determination is difficult and requires special investigations. Therefore, for solving problems of practical metrology, approximate methods of summation of errors (some of them are standardized, see, e.g., [4, 5]) are becoming widespread.

We shall consider the potentialities of approximate methods and the limits of their applicability.

It is known that summation of errors of direct measurements can be carried out arithmetically and geometrically by the formulas

\[
\Delta_A = \sum_{j=1}^{m} \Delta_j, \quad (1)
\]
\[
\Delta_G = \sqrt{\sum_{j=1}^{m} \Delta_j^2}, \quad (2)
\]

where \(\Delta_A\) is the arithmetic total error; \(\Delta_G\) is the geometric total error and \(\Delta_j\) is the j-th error component.

It is also known that the arithmetic summation overestimates while the geometric one underestimates the results, i.e. the actual value of the total error \(\Delta_{\Sigma}\) is situated in the interval given by the relation

\[
\Delta_G < \Delta_{\Sigma} < \Delta_A. \quad (3)
\]

Corrected values for the quantities \(\Delta_A\) and \(\Delta_G\) given by the relations

\[
\Delta_{\Lambda K} = k_1 \sum_{j=1}^{m} \Delta_j, \quad (4)
\]
\[
\Delta_{\Gamma K} = k_2 \sqrt{\sum_{j=1}^{m} \Delta_j^2}, \quad (5)
\]

(see [1-4, 6, 7]) can be used as point estimators of the total error. Here \(\Delta_{\Lambda K}\) and \(\Delta_{\Gamma K}\) are the corrected arithmetic and geometric total errors respectively; \(k_1\) and \(k_2\) are the coefficients (\(k_1 < 1, k_2 > 1\)).

Translated from Izmeritel'naya Tekhnika, No. 4, pp. 10-12, April, 1995.
TABLE 1

<table>
<thead>
<tr>
<th>$\Delta \Delta_\Sigma$</th>
<th>$\Delta_\Sigma$</th>
<th>$\Delta_\Gamma$</th>
<th>$\Delta_\Delta_\Sigma$</th>
<th>$\delta_{\Delta_\Sigma}$, %</th>
<th>$\delta_{\Delta_\Gamma}$, %</th>
<th>$\delta_{\Delta_\Gamma}$, %</th>
<th>$\Delta_{\Sigma_{\Gamma}}$</th>
<th>$\delta_{\Sigma_{\Gamma}}$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
</tr>
<tr>
<td>0.10</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
</tr>
<tr>
<td>0.25</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
</tr>
<tr>
<td>0.30</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
</tr>
<tr>
<td>0.70</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
</tr>
<tr>
<td>1.00</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
</tr>
<tr>
<td>3.00</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
</tr>
<tr>
<td>4.00</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
</tr>
<tr>
<td>10.0</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
</tr>
<tr>
<td>12.0</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
<td>1.090</td>
</tr>
</tbody>
</table>

The value of the coefficient $k_1$ is usually correlated with the preassigned confidence probability ($P_g$) and the relations among constituent errors [6, 7]. As a first approximation (in the case of two summands) the value $k_1 = 0.80$ is recommended for $P_g = 0.95$ and $k_1 = 0.85$ is suggested for $P_g = 0.99$.

The value of the coefficient $k_2$ depends on $P_g$ and the number of summands. According to [4] for $P_g = 0.95$ and for two or more summands the value of $k_2$ should be 1.1, while for $P_g = 0.99$ the coefficient (in the first approximation) takes on the value from 1.2 up to 1.4 depending on the number of summands.

In different cases summation of errors is carried out utilizing different formulas. Thus the summation of random and systematic error components of measuring devices is carried out by means of formula (1) [5], summation of independent random error components (more precisely summation of the mean squared deviations) is carried out geometrically by means of the formula (2) [1-3], and summation of uneliminated systematic error components of the measurement results is performed using formula (5) [4].

Summation of random and systematic components of measurement errors can be carried out by means of formula (4) [6, 7] or in accordance with [4] using

$$\Lambda = k_2 \cdot \frac{\Sigma}{\Sigma}.$$  (6)