Determining the amount of information at the output of a digital angle converter

V. G. Domrachev and A. B. Skripnik

An information approach to estimation of precision of digital angle converters is proposed. The method is based on estimating the amount of information of individual code combinations. It is shown that an analytic expression for the density distribution function of information can be obtained.

Informational methods of estimating metrological characteristics of digital angular converters (DAC) are not widespread at the present time. This is due to the absence of precise definitions of information in relation to measurement technology and the terminological confusion resulting from this state of affairs. Also, it is quite difficult to perceive the results of informational estimation of metrological characteristics when only insufficient specialized knowledge in the field of information theory is available. In spite of these obstacles, continuation of the research in this direction allows us to obtain interesting results, and informational methods under investigation may become an additional tool for solving a number of metrological problems.

The propositions developed by C. E. Shannon serve as the basis for the majority of the well-known methods of estimating metrological characteristics of digital measuring transformers. Shannon proposed a single parameter — called entropy — for the distribution law which is a weight function of all the points of the distribution curve. The main problem which arises in utilization of such an approach is that the entropy is a moment of a centered random error component. Here the systematic error component ought to be taken into account separately which interferes with the overall use of the information criterion. Another deficiency of these methods is the fact that the value of information obtained in the manner described above is just a mean value which is insufficient for a deeper description of precision capabilities of measuring transformers. Thus utilization of entropy in metrology is not very efficient.

Below we shall propose an approach to determination of the amount of information which is free of these defects. The essence of the method is that the basic notions of information theory are applied to individual coding combinations. In the course of the development of the method the groundbreaking works [1, 2] were utilized as well as a number of results obtained in [3].

Consider the order of determination of information contained in a specific coding combination of DAC. It is known that the experts who utilize DAC when estimating the precision of specific coding combinations are interested mainly in the total error of transformation. Thus an application of information theory to estimation of precision associated with a realization of a specific coding combination of a DAC can be viewed as implementation of a procedure to narrow down the uncertainty interval with respect to the calculated value of the angle within which the true value of the measured angle is uniquely situated.

We shall explain this by means of an example. It is known that the k-th coding combination at the output of a DAC is perceived in the systems of information processing as a specific value of an angle defined as \( \varphi_k = Kq \), where \( \varphi_k \) is the calculated value of the angle corresponding to the k-th coding combination, \( K \) is the index of the coding combination for \( K = 0 \ldots (2^N - 1) \), where \( N \) is the number of the digits in the DAC, and \( q \) is the quantization step of the DAC.

For an ideal DAC the value \( \varphi_k \) corresponds to the middle of the calculated k-th quantum and the interval in which the true value of the measured angle is located does not exceed the value \( \pm 0.5 \ q \). However, the coordinates of the limits of real quanta seldom coincide with the calculated ones. Therefore, it is impossible to determine beforehand the interval in which the true value of the measured angle with respect to the value of \( \varphi_k \) is located. This interval can be determined only after measuring the actual coordinates of the shift of the k-th coding combination. In the case when the actual k-th quantum is shifted to the left, the maximal error which can occur is determined to be \( e_{k_{\max}} = 0.5q + |e_k| \), and when shifted to the right as \( e_{k_{\max}} = \frac{1}{2} \).

Fig. 1. Dependence of the information at the output of a DCA on the error of reproducing quantization levels.

Fig. 2. A histogram of a sample of information values at the output of a specimen of the transformer:angle-parameter code.

0.5q + |ε_k|, where ε_k is the corresponding error for the end-point of this quantum. The reproduction error of the quantum level is meant to be the error of the coordinate of the shift of the k-th coding combination. The value of the error ε_kmax corresponds to the one-half of the uncertainty interval Δ_k within which the true value of the measured quantity with respect to the calculated value φ_k is located. The quantity Δ_k is determined as φ_k.

Thus the range of measuring the angle Q and the interval Δ_k are the values of the uncertainty intervals before and after the errors ε_kl and ε_kp were measured. In this case the amount of information about the interval Δ_k will be (in bits):

$$I_k = \log_2 \left( \frac{Q}{\Delta_k} \right).$$

(1)

The relative error ε equals the ratio Δ_k/Q, is the defining one from the aspect of the information under consideration. This representation of information actually coincides with the well-known Hartley measure.

Explicitly expression (1) can be written as

$$I_k = \log_2 \left( \frac{Q}{2 \cdot (0.5q + |ε_k|)} \right).$$

(2)