IS IT POSSIBLE TO REFINE A NONEXISTING STANDARD?

M. F. Yudin

The new suggested definition of the unit of the amount of substance in terms of the inertial and gravitational properties of the xenon mole suggested in [1] is briefly examined. The fallacy of this suggestion is shown.

The Eleventh General Conference on Weights and Measures has established the International System of Units (SI) and recommended it for general use in all branches of science and engineering. A physical quantity called the amount of substance was introduced in 1971 as a basic unit of this system [2]. Unfortunately, up to now no international metrological organization gave a clear definition of this quantity. This is also true of the document [3] jointly worked out by seven international organizations: the International Bureau of Weights and Measures (BIPM), the International Federation of Clinical Chemistry (IFCC), the International Unions of Pure and Applied Physics and Chemistry (IUPAP and IUPAC), and International Organization of Legislative Metrology (OIML). The application of this physical quantity frequently led and still leads to misunderstandings. The latest example is the paper [1] whose shortcomings are discussed below.

The mole, recommended by the SI a unit of measurement of the amount of substance, is defined as the total number of particles (atoms, molecules, ...) of substance of the considered system equal to the number of atoms contained in 12 g of the $^{12}$C nuclide. According to contemporary concepts, the number of atoms in 12 g of the $^{12}$C nuclide is

$$N_A = (6.0221367 \pm 0.0000036) \times 10^{23} \text{ atoms/mole}$$

and is called the Avogadro number [4].

Thus, the mole is actually defined as the number of "elementary" particles and is related to carbon mass of nuclide $^{12}$C. The value of $N_A$ will change with any refinement of the mass of the $^{12}$C atom.

The shortcomings of such a concept of the amount of substance and of its unit were discussed in detail in 1970 [5]. In that paper we mentioned the so-called law of equivalents used in the study and calculations of various chemical reactions. According to this law different chemical elements combine in amounts by weight proportional to their equivalents. As is known, by the equivalent of any chemical substance is meant the ratio of the atomic mass of this element to its valence. Accordingly, a chemical measure of the mass of substance called the kilogram-molecule (gram-molecule), or briefly kilomole (mole) is frequently used in chemical calculations besides the unit of mass kilogram (gram). It was noted in [5] that:

if the amount of any substance is characterized by its mass, the kilomole as a unit of the amount of substance will be different for different substances (different nuclides);

if by the amount of substance is meant the number of atoms (molecules) constituting it, in a closed system the number of molecules in decomposition or synthesis reactions will vary and, according to the definition of mole, so will the amount of substance. Consequently, also here the kilomole (mole) cannot be recommended as a unit of the amount of substance.

Thus, even if the kilomole, characterized by the mass of substance, is convenient for chemical calculations, it has a different size for different substances (nuclides) and cannot be considered as a unit of any consistent system of units including the SI. The physical concept of "the amount of substance" should be excluded from the list of basic USI quantities and the mole, as a special unit of mass convenient for chemists, should be allowed for equal use with other SI units, as was done for the special unit of mass of atoms and molecules by introducing a unified unit of mass $m_u$ equal to 1/12 the rest mass of a neutral atom of $^{12}$C nuclide in ground state:

$$m_u = (1.6605402 \pm 0.0000010) \times 10^{-27} \text{ kg}.$$
It should be noted that the concept of the amount of substance as a characteristic of matter is associated with the assumption of atomic structure of matter. The number of particles, without any connection to the mass of a single particle, first played a role in the equation of state of an ideal gas whose pressure $p$ obeys the Mendeleev-Clapeyron equation

$$pV = nRT,$$

where $V$ is the volume occupied by the gas, $n$ is the number of gas moles, $R = kN_A$ is the universal gas constant, $T$ is the absolute temperature, and $k$ is the Boltzmann constant.

Thus, the amount of substance measured in moles was accepted as a basic quantity in the SI.

M. G. Kozlov [1] proposed to define the unit of the amount of substance as the amount of any substance that has gravitational and inertial properties the same as those of pure xenon..." To understand the fallacy of this suggestion we have to turn to Newton who introduced to classical mechanics the concept of mass as a measure of inertia and gravitation of bodies. According to classical mechanics the mass of a body is a fundamental characteristic of matter that determines its inertial and gravitational properties. In the definition of momentum of a body the momentum $P$ is proportional to the velocity of free motion $v$:

$$P = mv,$$

where the factor of proportionality $m$ is a constant for a given body and represents its mass.

An equivalent definition of mass follows from the equation of motion of the classical mechanics of Newton:

$$f = ma.$$  \hspace{1cm} (2)

Here the mass $m$ is the factor of proportionality between the force $f$ acting on a body and the acceleration $a$ produced by it. Mass thus defined is a characteristic of the body properties (the greater the mass of a body the less the acceleration it acquires under the action of a constant force) and is called the inertial or inert mass.

In Newton’s theory of gravity mass is regarded as a source of the field of gravity. Every body creates a gravitational field and is acted upon by the gravitational field created by other bodies whose force also is proportional to mass. According to Newton’s law of gravitation

$$f = Gm_1m_2/r^2,$$  \hspace{1cm} (3)

where $G$ is the universal gravitational constant, and $m_1$ and $m_2$ are the masses of mutually attracting bodies.

The masses of bodies defined by expression (3) are called gravitational masses.

Experience shows that the inert and gravitational masses are numerically equal. This is a fundamental law of nature and is called the principle of equivalence. It should be stressed that at first Newton considered the mass of a body as a measure of the amount of substance, which is still quite true in case of homogeneous (mononucleid) bodies.

In relativistic mechanics the relation between momentum and particle velocity is given by

$$P = m_0v\sqrt{1 - v^2/c^2}$$  \hspace{1cm} (4)

where $m_0$ is the rest mass, and $m = m_0\sqrt{1 - v^2/c^2}$ is the mass of a moving body that depends on its velocity, i.e., the mass of a particle increases with its velocity.

If $v \ll c$, expression (4) turns into (1).

According to the theory of relativity, the mass of a particle is related to energy by

$$e = mc^2 = m_0c^2\sqrt{1 - v^2/c^2}.$$  \hspace{1cm} (5)

The rest mass $m_0$ determines the internal energy of a particle, the so-called rest energy $e_0 = m_0c^2$. Energy is thus always associated with mass. Therefore in relativistic mechanics the laws of conservation of mass and energy are not separate but are