GENERAL PROBLEMS IN METROLOGY AND MEASUREMENT TECHNIQUES

OPTIMIZATION OF MEASUREMENT PROCESSES
UNDER METROLOGICAL PROVISION FOR COMPLEX TECHNOLOGICAL SYSTEMS

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A dynamic problem of optimizing a selection of measured quantities and measuring devices is formulated. For a class of linear dynamic objects, an optimal controller of the measurement processes is designed. An example of solving an optimization problem for metrological provision of a testing equipment is presented.

The problem of selecting a list of measured quantities, measurement devices and the sample size of the measured information arises in the course of experimentation, solution of production problems, design, testing and running of technological systems. It accompanies the scientific technological progress at all the stages of industrial development. Galileo's dictum: "Measure everything that is measurable and make it accessible to everything which cannot be measured" remains an absolute truth for dozens of generations of engineers and scientists. However, at each stage of technological development, in each specific area of investigations, the "accessibility of measurements" is interpreted in a different manner. An economic approach in some form has always been the basis for a solution of the problem.

At the current stage of technological development, to decide whether it is expedient to measure certain quantities, various modifications of economic criteria are utilized which can be reduced to the following two groups. The first contains criteria based on minimization or a comparative analysis of the expenditures involved in carrying out the corresponding metrological activities and of the losses incurred in the realm of utilization of their results. This approach is closely associated with the production efficiency and has become quite widespread in industry, in particular in design of an automated production [1, 2]. The second group of criteria is aimed, to a large extent, at the problem of metrological provision of technological objects utilized in defense industry problems; personal security, ecology and medicine. A special feature of these criteria is taking into account the effect of metrological provision of systems on the efficiency of their applicability, for example via indices of the probability of fulfillment of some basic functions by a system [3, 4].

In spite of the fundamental difference in the approach of constructing the objective function, implementation of the above stated criteria has common features. Independently of the selected criterion, the accounting for the dynamics of the state of an object of measurements and the metrological properties of a measuring equipment is carried out on the basis of the so-called "guaranteed" approach. This means that the probabilistic analysis of the dynamics of the behavior of an object and the drift of the errors in measuring devices is reduced to an estimation of the maximal parameters of these stochastic processes. Based on these estimates, periods for testing appliances are assigned and knowingly redundant measurement programs are formed. Taking into account the differences between the individual and normative characteristics of objects and devices allows us to substantially reduce the expenditures related to measurement of specific objects without a detriment.

At present time, a technological foundation for solution of this problem has actually been created. The development of informational technology resulted in widespread penetration of computational hardware and software into automated technological processes and complex engineering systems. Creation of modular unified control-testing systems based on modern microelectronics allows us to perform this task without substantial organizational and financial difficulties. An integrated automation of testing activities enable us to analyze in detail the dynamics of errors of a group of measuring devices and even of each individual apparatus [5].

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However, if the technical aspect of the problems of adaptive control of measurement processes has been solved by the well-known methods, mathematical foundations for measurement optimization still require further development. In this paper a solution of the problem of optimizing measurements based on design and solution of the problem of optimal control of measurement processes in partially-observable complex dynamic systems is proposed.

The technological state of an object of observations is determined by the totality of varying properties which are characterized at a given instant of time by the values of the parameters of an object. These parameters are connected by a probabilistic spatial—temporal dependence. The mathematical model of the state of an object can be presented by a vector difference equation of the first order

\[ \hat{U}_{r+1} = \Phi_{r+1/r} \hat{U}_r + B_{r+1/r} W_r, \]

where \( \hat{U}_{r+1}, \hat{U}_r \) are \( n \)-vectors of the state at times \( r + 1 \) and \( r \); \( \Phi_{r+1/r}, B_{r+1/r} \) are the state transition matrices of order \( n \times n \); \( W_r \) is a random \( n \)-vector of normally distributed disturbances.

The state of an object under consideration is observed at times \( r \) which correspond to the times at which certification was performed. The measurement model will be represented in the form

\[ I_r = \alpha_r (C_r + F_r U_r) + V_r, \]

where \( I_r \) is the vector of parameters (a result of estimation of the state parameters); \( \alpha_r \) is a diagonal matrix of dimension \( m \times m \) whose elements \( \alpha_{r}^{i,j} \) (\( i, j = 1, m \)) take on values 0 or 1:

\[ \sigma_{r}^{i,j} = \begin{cases} 0 & \text{if the parameter } U_r^{i} \text{ is not measured;} \\ 1 & \text{if the parameter is measured.} \end{cases} \]

\( C_r \) is a deterministic \( m \)-vector corresponding to the operational conditions;
\( F_r \) is a design matrix of dimension \( m \times n \) which connects the vector of parameters with the state vector;
\( U_r \) is an \( n \)-dimensional vector of measurements corresponding to the measured information at the output of the MS;
\( V_r \) is a random \( m \)-vector of the results of the effect of disturbances on the measuring device which is normally distributed.

The optimal Kalman filter [6] serves as the basis of the computational scheme of the algorithm for obtaining the state estimators. One of the basic advantages of the filter is the possibility of implementing a recursive processing of the measurement information in the real time scale, and to obtain an estimator of the current or a forecasted state of the object immediately after the successive result of measurements has been obtained. However, in this case, this property of the filter is not decisive. When choosing an algorithm for obtaining state estimators, the following properties of the Kalman filter were taken into account:

- the filter uses a convenient computational procedure and provides a form of an adequate presentation of the dynamics of random processes which includes in a natural manner errors in description of these processes by means of mathematical models;
- in view of the assumptions on the models (1) and (2), the estimator obtained by means of a Kalman filter will be the most efficient in the framework of the description of the state process under consideration;
- estimation of the state of an object is carried out taking the prehistory of the behavior of the state parameters into account, which allows us to utilize prior information concerning the object including the results of previous certifications.

The computational procedure of an algorithm of the Kalman filter as applied to the problems of measurement control is considered in detail in [7]. The following problems are solved when determining the appropriateness of the algorithm to the real-world conditions:

- null-hypothesis concerning the distributions of the quantities \( W_r \) and \( V_r \) are tested;
- the form of the estimated parameter — the characteristic of the behavior of the covariance matrix of the estimation errors of the filter \( K_{r/r} \) in time in the form of its trace is determined;
- the form of representing of the initial data — that of components of the matrices \( U_r, K_{r/r}, r = 0 \) is determined;
- the form of representing the results of estimation \( T_r(K_{r/r}) \) is determined; and also the trajectory of the trace of the matrix \( K_{r/r} \) in time \( r \in [0, T] \).