Utilization of the Error Entropy Value as a Criterion of the Precision of Instruments and Measurements

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It has been shown in [1] that the application of either the usual or the latest methods for normalizing the precision of measuring devices amounts to finding one or several values of the absolute or relative error (coefficients in one, two or three-term error formulas). In an experimental determination these values are represented as particular realizations of a random quantity which can be fully described only by indicating its probability distribution law, or more concisely finding its limiting, root-mean-square or entropy [2] value.

However, it has been shown in [2, 3] that neither the root-mean-square, nor the limiting error values determine unambiguously the information content obtained by measurements. If we assume as an incontestable truth that the main value of measurements consists in their information content about the measured quantity, and the aforementioned criteria are not related unambiguously to the information, doubt is naturally cast (at least in its theoretical aspect) on the justification for using these criteria in evaluating the precision of instruments and measurements.

If at the same time it is known that there exists a criterion in the form of the error entropy value [2] which is related unambiguously to the measured information content, it is only natural that there should arise the question whether it is possible and advisable to use in practice this criterion and discontinue the application of the other criteria.

The possibility of using the error entropy value as the basic criterion in evaluating the precision of instruments and measurements depends above all on the existence and practical applicability of methods for determining this error value. The problem of the advisability of changing over to the error entropy value for evaluating instruments and measurements has an important theoretical significance. The fact of the matter is that the concept of an error serves as a foundation for all the legislation in the field of instrument making, for all the standards, norms, rules for the acceptance and certification of instruments, etc. If from the point of view of the information theory, i.e., from the point of view for which the properties of the measuring equipment are most vitally important (like the energy characteristics for the energy-measuring devices) the existing legislation is insufficiently substantiated theoretically, the problem arises about the advisability and necessity of reviewing the legal regulations. However, it is necessary to discriminate carefully between the things which should be altered and those which need not be changed if the error entropy value is taken as the basic criterion of the precision of instruments and measurements.

A practical evaluation of the error entropy value can be made on the basis of its definition [2], i.e., according to the relationship

\[ H(X) = - \int_{-\infty}^{+\infty} p(x) \ln p(x) \, dx \]  

and

\[ \Delta = \frac{1}{2} e^H(X), \]  

where \( p(x) \) is the error probabilities distribution density, \( \Delta \) is the entropy value of this error.

However, in evaluating the precision of measuring devices or measurements the actual error distribution law \( p(x) \) is not available, with the exception of a certain finite number \( n \) of specific random quantity values which follow this law. From this limited number of results it is possible to plot only a stepped histogram (Fig. 1) which approaches to a given extent the actual distribution law \( p(x) \). Therefore, in practice it is desirable to have a formula for calculating the error entropy value from such an approximate histogram.

If the histogram consists of \( m \) columns with boundaries \( X_0, X_1, X_2, \ldots, X_i, X_{i+1}, \ldots, X_m \) and each column has a width of \( d_i = X_{i+1} - X_i \) and comprises \( n_i \) results, the probability density along each column remains constant and equal to

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\[ p(x) = \frac{n_i}{n \cdot d_i} \]

The entropy of such a stepped distribution is

\[
H(X) = -\int_{-\infty}^{+\infty} p(x) \ln p(x) \, dx = -\sum_{i=1}^{m} \frac{n_i}{nd_i} \ln \frac{n_i}{nd_i} \, dx = \sum_{i=1}^{m} \frac{n_i}{n} \ln \frac{n_i}{n}.
\]

If the width of all the histogram columns is the same, i.e., if \(d_i = d\), we find that

\[
H(X) = \ln d + \sum_{i=1}^{m} \ln \left( \frac{n_i}{n_i} \right) = \ln \left[ d \prod_{i=1}^{m} \left( \frac{n_i}{n} \right)^{n_i} \right],
\]

and then according to (3) the error entropy value will be equal to

\[
\Delta = \frac{1}{2} e^{H(X)} = \frac{d}{2} \prod_{i=1}^{m} \left( \frac{n}{n_i} \right)^{n_i} = \frac{d}{2} \cdot \sqrt[n]{\prod_{i=1}^{m} (n_i)^{n_i}}.
\]

Investigation (5) shows that the evaluation thus obtained is displaced. This displacement it would appear decreases with rising values of \(n\) and \(m\). Moreover, \(m\) and \(d\) can be selected arbitrarily with the limitation that \(n_i \geq 2\), since the existence of an interval with \(n_i = 0\) or \(n_i = 1\) does not affect the entropy because \((n_i)^{n_i} = 0^0 = 1\) and \(1^1 = 1\).

An approximate method for evaluating the error entropy value can be based on the relationship \(\Delta = K \cdot \sigma\), where \(K = \Delta / \sigma\) is the entropy coefficient [2], and \(\sigma\) is the root-mean-square value of the error. In order to find the range of \(K\) values we calculated it from the experimental data obtain in testing 47 measuring devices (19 moving-coil, electrodynamic and moving-iron instruments with pivots; 21 electronic instruments for measuring temperature, efforts and emfs; and 7 digital instruments of the type KL-48 and R-306). From a series of measurements with \(n = 50-300\) we calculated the value of the entropy error, the entropy coefficient and the square root of the inverse quantity of the relative fourth moment. The results of these measurements are shown in Fig. 2 with the squares denoting pointer instruments, the circles—electronic instruments, and the diagonal crosses—digital instruments. The same graph carries the theoretical curve of \(K\) obtained from [3], where points 1 and 2 indicate the coordinates respectively of a normal and uniform distribution.

This investigation leads to several interesting conclusions. In theoretical investigations of measuring devices it is assumed that their errors have a normal or a uniform distribution (respectively point 1 and 2 in Fig. 2). In fact, however, we find that the error distribution laws of measuring devices shown in rectangular coordinates on Fig. 2 occupy an area extending far to the left and down from the line which connects points 1 and 2, i.e., the errors can have