FORECASTING AND PROVIDING RELIABILITY IN MEANS OF
MEASUREMENT ON THE BASIS OF EXPLICIT AND LATENT FAILURES

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On p. 174 of [1], a model was examined for a binary system consisting of two series-connected parts 0 and 1, in which failure in part 0 was indicated, while failure in part 1 was detected only in special checks. The reliability functions are correspondingly $P_0(t) = \exp(-\lambda t)$ and $P_1(t)$ arbitrary. That model for example gives a good description of a measuring instrument or system in which one of the devices is mainly subject to metrological failures, i.e., ones of latent type, while the other shows explicit failures that prevent the system from operating.

In [1] however, an exponential form was proposed for $P_1(t)$ for cases where checking was envisaged (in contrast to obligatory replacement).

The [2, 3] approach for deriving reliability characteristics in periods between checks can eliminate that constraint with the following strategy for servicing the 0–1 system:

1) part 1 is immediately replaced if found to be faulty on check;
2) failed part 0 is replaced on the next check;
3) the system is not used up to the next check after part 0 has failed; and
4) the intervals between checks should be identical at $\tau$.

That situation is characteristic of a batch of identical binary systems or set of measuring instruments showing two independent failure fluxes in the absence of a checking and repair service at the point of use.

We introduce symbols for this system with $k \geq 0$: $A_t^{k+1}$ the absence of a failure in part 1 during time $t$ reckoned from check $k$, $B_t^{k+1}$ the absence of failure in part 0 during $t$ reckoned from check $k$, $C^k = A_t^k$ the retention of part 1 in a viable state on check $k$, and $\epsilon_k = 1 - P(C^k)$; $P_{k+1}(t) = P(A_t^{k+1} \mid B_t^{k+1})$. The zero check is made at the initial instant.

We derive a recurrent relation for $P_{k+1}(t)$. For $k \geq 1$

$$P_{k+1}(t) = P(A_t^{k+1} \land C^k \mid B_t^{k+1}) + P(A_t^{k+1} \land C^k \mid B_t^{k+1}).$$

Standard formulas in probability theory give

$$P(A_t^{k+1} \land C^k \mid B_t^{k+1}) = P(A_t^{k+1} | C^k \land B_t^{k+1}) P(C^k | B_t^{k+1}).$$

As $C^k$ and $B_t^{k+1}$ are independent and failed parts 1 are completely replaced on check $k$,

$$P(A_t^{k+1} \land C^k \mid B_t^{k+1}) = P_t(t) \epsilon_k.$$

We consider the interval between checks $k - 1$ and $k$ and use the feature of an exponential distribution in which the check instant can be taken as the origin for the viability time for part 0. Then as $f_0(t) = -P_0'(t) = \lambda \exp(-\lambda t)$, we get from the formula for the total probability that

$$P(A_t^{k+1} \land C^k \mid B_t^{k+1}) = \int_0^\tau \lambda \exp(-\lambda x) P_k(x+t) dx + \exp(-\lambda \tau) P_k(\tau + t).$$

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Then

\[ P_{k+1}(t) = \int_0^\tau \lambda \exp(-\lambda x) P_k(x) \, dx + \exp(-\lambda t) P_k(t) + \varepsilon_k P_k(t), \]

(1)

where

\[ \varepsilon_k = 1 - \int_0^\tau \lambda \exp(-\lambda x) P_k(x) \, dx - \exp(-\lambda t) P_k(t). \]

(2)

We integrate (1) and (2) by parts and substitute (2) into (1) to get

\[ P_{k+1}(t) = \int_0^\tau \exp(-\lambda x) P_k(t+x) \, dx + \int_0^\tau \exp(-\lambda x) P_k(t-x) \, dx + \varepsilon_k P_k(t), \]

(3)

or in operator form

\[ P_{k+1}(t) = (E-I+W)[P_k(t)], \]

(4)

in which \( E[\varphi(t)] = \varphi(t) \) is the identity operator and

\[ I[\varphi(t)] = \int_0^\tau \exp(-\lambda x) \varphi(t+x) \, dx; \quad W[\varphi(t)] = -\int_0^\tau \exp(-\lambda x) \varphi'(x) \, dx. \]

We show by induction that for \( k \geq 0 \)

\[ P_{k+1}(t) = \sum_{i=0}^k \varepsilon_{k-i} (E-I)^i [P_i(t)], \]

(5)

in which \( \varepsilon_0 = 1 \) and the zero degree of the operator is the identity one.

One can check (5) directly for \( k = 0 \).

One can operate with polynomials containing \( I \) and \( E \) in the same way as with ordinary ones because these linear operators commute, in contrast to the operators \( I \) and \( W \), and we can substitute \( P_k(t) \) from (5) on the right in (4).

Then we get (5) together with the following from (3) for \( j \geq 1 \):

\[ \varepsilon_j = \sum_{i=0}^{j-1} \varepsilon_{j-1-i} \{ I(E-I)^i [P_i(t)] \}_t=0. \]

(6)

Other reliability characteristics can be derived successively for each interval from (5) and (6), e.g., the probability of fault-free operation for the system as a whole before check \( k+1 \):

\[ P_{k+1}^0 = \exp(-\lambda t) \sum_{i=0}^k \varepsilon_{k-i} (E-I)^i [P_i(t)]_t=0. \]

Example. A batch of identical binary systems is in use, with the parts 1 derived from two plants in the ratio \( c_1 : c_2 \) (\( c_1 + c_2 = 1 \)) and correspondingly having failure parameters \( \lambda_1 \) and \( \lambda_2 \), while the 0 parts are provided with characteristic \( \lambda \) by a single plant. The systems are serviced on the above strategy. One has to establish the maximum interval between checks, which must be such that the mean proportion of systems in a state of failure does not exceed \( \delta_1 \) at any instant and the mean proportion of replaced 0 parts at each check does not exceed \( \delta_2 \), while the mean proportion of replaced 1 parts at each check does not exceed \( \delta_3 \).

The task is as follows from the viewpoint of the above model.

Find \( r_0 = \max r \) subject to the constraints

\[ \exp(-\lambda t) P_{k+1}(t) > 1 - \delta_1, \quad k > 0, \quad t < \tau; \]

(7)