MEASUREMENT OF DIFFERENTIAL PARAMETERS OF CHARGED PARTICLE BEAMS

A. B. Gaiduchenko and V. M. Rybin

Reconstruction of differential parameters of a beam of charged particles from information on integral parameters obtained from probes in the form of wires with a diameter much less than the beam parameter is considered. A convolution and inverse projection algorithm (for parallel and cone scanning) and a Fourier algorithm are discussed as well as is the measurement accuracy and resolution as functions of different factors and demands.

In practice one frequently has to know the distribution density of a certain physical quantity in space. These problems are frequently encountered in medicine (computer tomography), astronomy (image reconstruction of distant celestial bodies), geophysics (mapping of rock structure), study of materials (investigation of internal structure), etc., [1-3].

Here we consider the possibility of reconstruction of the density distribution function of current across a beam of charged particles. The density distribution function of a current of charged particles across the beam is referred to as the linear beam profile in this cross section. Profiles can be differential or integral [4, 5].

Vertical and horizontal differential beam profiles are described by

\[ P_{zy}(x) = j(x, y, z_0, t) \bigg|_{z=z_0} \quad \text{and} \quad \]
\[ P_{zx}(y) = j(x, y, z_0, t) \bigg|_{x=x_0} \]

where \( j \) denotes three-dimensional current density.

An integral profile is obtained by integrating a differential one with respect to one coordinate while the other remains fixed. The respective integral profiles are given by

\[ P_{z0}(x) = \int_{y} j(x, y, z_0, t) dy = j_{z0}(x, t) \]
\[ P_{x0}(y) = \int_{x} j(x, y, z_0, t) dx = j_{x0}(y, t) \]

An integral beam profile can be measured with the aid of wire probes. These profiles are then used to reconstruct the current distribution density function \( j(x, y) \) across the beam, i.e., to obtain its differential characteristics.

Reconstruction Algorithm. Let us consider the general problem of reconstructing an object image from its projections [1, 3]. Consider Fig. 1. The Cartesian coordinates \((u_1, u_2)\) and \((\hat{u}_1, \hat{u}_2)\) are related by

\[ u_1 = u_1 \cos \theta + u_2 \sin \theta; \]
\[ u_2 = -u_1 \sin \theta + u_2 \cos \theta. \]

A photon beam passes through a material with a homogeneous structure with a distribution \( x(u_1, u_2) \). The beam intensity varies exponentially as

\[ I(u_1) = I_0 \cdot e^{-\int x(u_1, u_2) du_2}. \]
The projection of \( x \) at an angle \( \theta \) is

\[
P(\hat{u}_1) = -\ln[I(\hat{u}_1)/I_0(\hat{u}_1)] = \int_{-\infty}^{\infty} x(\hat{u}_1 \cos \theta - \hat{u}_2 \sin \theta; \hat{u}_1 \sin \theta + \hat{u}_2 \cos \theta) \, d\hat{u}_2.
\]

The similarity of this expression and the expression for interrelation between the differential and integral beam profiles when \( u_1 = x, u_2 = y, P_{20}(x) = P_1(\hat{u}_1), \theta = 0 \) is noticeable.

Let us apply the Fourier transform (the index 2 indicating two-dimensional transformation)

\[
S_\theta(\omega) = \mathcal{F}[\rho_\theta(\hat{u}_1)];
\]

\[
X(\Omega_1, \Omega_2) = \mathcal{F}_2[x(u_1, u_2)];
\]

\[
S_\theta(\omega) = \int_{-\infty}^{\infty} \rho_\theta(\hat{u}_1) \exp[-j\omega(\hat{u}_1 \cos \theta)] \, d\hat{u}_1, \quad j = \sqrt{-1}.
\]

In the original system of coordinates

\[
S_\theta(\omega) = \mathcal{F}[x(u_1, u_2) \exp[-j\omega(\hat{u}_1 \cos \theta + \hat{u}_2 \sin \theta)] \, d\hat{u}_1 \, d\hat{u}_2.
\]

From the central projection theorem follows

\[
S_\theta(\omega) = X(\omega \cos \theta; \omega \sin \theta).
\]

Here \( X \), the cross section of the two-dimensional Fourier transform of \( x(u_1, u_2) \) at an angle \( \theta \), is a one-dimensional Fourier transform of the projection \( p_\theta(\hat{u}_1) \) at the angle \( \theta \). Thus, \( x(u_1, u_2) \) can be found as a two-dimensional inverse Fourier transform:

\[
x(u_1, u_2) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(\Omega_1, \Omega_2) \exp[j\Omega_1 u_1 + j\Omega_2 u_2] \, d\Omega_1 \, d\Omega_2.
\]

Passing in the two-dimensional spectral density to polar coordinates \( (\omega, \theta) \), we get

\[
x(u_1, u_2) = \frac{1}{4\pi^2} \int_{0}^{\pi} \int_{0}^{\infty} S_\theta(\omega) \exp[j\omega(\hat{u}_1 \cos \theta + \hat{u}_2 \sin \theta)] \, d\omega d\theta,
\]

where the internal integral is an inverse one-dimensional Fourier transform of the product \( S_\theta(\omega) | \omega | \).

It was proved in [1, 2] that the solution of the problem in the space domain can be written as