A new method for estimating precision of analog—digital devices based on the entropy approach is proposed. A special feature of the method is its sensitivity to a constant error component.

The method considered below can be utilized for any analog—digital devices which are characterized by the presence of an analog parameter at the input and a digital code at the output. Among these are analog—digital transformer, digital angle transformers (DAT), linear displacements, transformers, etc.

The possibilities and special features of the method are considered by means of an example of a large class of devices — DAT. It is known that in these devices the precision of transforms is estimated based on deterministic and probabilistic criteria. Moreover, recently integral precision criteria were introduced and developed and information-oriented criteria occupy a significant place among them.

When utilizing DAT as a part of microprocessor and computerized control systems, the most interesting are precision criteria associated with the informational estimators. Such an approach allows us to simultaneously carry out precision optimization of analog as well as digital components of systems. Advantages of the information approach to precision estimation of measurement systems are described in [1-6].

In practice, the information approach is more often utilized in systems of information transmission rather than for measurement systems. This is due to the fact that the entropy being a numerical estimator of uncertainty measure does not reflect the effect of the constant component of the error (since it does not possess uncertainty). Therefore, in many cases when performing calculations, other criteria models are preferred; their simplicity and clarity being one of the main reasons. The entropy approach is fundamentally different from other formal criteria estimators since it is connected with the notion of information content and is, therefore, of a substantive nature. However, in order to apply the entropy precision criterion to DAT it is necessary to analyze as far as possible the structure of errors and their sources.

The cumulative error of DAT without accounting for the external factor is composed, as a rule, from the quantization error \( E_1 \) and the instrumental error \( E_2 \) (which is the error of reproducing the quantization levels). In Fig. 1 the structure of the error of such a transformer is presented.

For a DAT with an ideal dial (ID) only the quantization error \( E_1 \) is present; in a real-world DAT the limits of the information bits of the real dial (RD) may be shifted in the space with respect to RD which generates the instrumental error \( E_2 \).

The quantization error for an N-digital DAT is a random variable which in the great majority of cases is uniformly distributed in the interval \( \pm q/2 \) where \( q = 2\pi 2^{-N} \) is the information bit the lower-order digit of the DAT (in radians). It uniquely assigns the upper information limit which can be reached as the result of the transformation. It is convenient to call this quantity the maximal information capacity of a DAT. In the case of an N-digital DAT it evidently constitutes N bits.

We pose the problem of determining the information capacity of a transformer taking the error \( E_2 \) into account. For a given quantization condition the required quantity is located in the limits from 0 to N bits and is equal to N when \( E_2 \) is absent (an ideal transformer) and it is equal to 0 if the digital code at the output of a DAT does not depend on the values of the angle at its input.

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In a general form \([4,7]\), \(J = N - H\), where \(J\) is the information capacity of the transformer in the presence of an instrumental error \(E_2\); \(H\) is the loss of information due to \(E_2\) or the entropy of the probability distribution law of the instrumental error.

Thus the problem of determination of information capability of a transformer reduces to calculation of the entropy of the probability distribution law of the error \(E_2\). If the distribution law is known then \(H\) can be obtained analytically:

\[
H = \int_0^{2\pi} p(E) \log_2 p(E) dE,
\]

where \(p(E)\) is the density function of the probability distribution of the error \(E\). It can be shown that the value of \(H\) remains unchanged when the constant component of the transformation error is present (a constant shift of the values of the angle corresponding to the output code relative to the value of the angle inputed into DAT).

Let the shift in the values of the angle be \(M\) radians, then

\[
H = \int_0^{2\pi} p(E-M) \log_2 p(E-M) dE = \int_M^{2\pi-M} p(E) \log_2 p(E) dE = \int_0^{2\pi} p(E) \log_2 p(E) dE.
\]

There is no contradiction in the result obtained. This is due to the fact mentioned above that the entropy possesses a specific property of a measure of uncertainty: the constant component does not affect the measure of uncertainty. From the aspect of transformational techniques only the absolute value of the error obtained as the result of single transformation is of importance. In a single experiment there is no possibility to collect data and to determine the constant component of the error. This might be the reason why the entropy approach has not received wide practical applications for estimation of the precision of DAT.

To utilize the entropy approach we ought to consider each transformation as an independent one and determine the loss of information of a DAT for the single transformation. This is feasible when, for example, one considers the contribution of each one of the digits of the DAT to the aggregate error separately and determines the amount of information which each digit loses at each transformation and then determine the mean value over several cycles of the control. Thus the problem reduces to determination of the amount of the lost information for each one of the digits of the transformer, under the condition that the probability \(s\) of obtaining an erroneous code in the digit under investigation is already known from the statistics of the trials.

We shall view the loss of information under transformation for a single digit as the conditional entropy \(H(Y|X)\) where the system \(Y\) is the totality of all the codes obtained after the transformation and \(X\) is the totality of codes corresponding to the values of the angle inputed at the single digit transformer. Thus the conditional entropy \(H(Y|X)\) represents the entropy of the system under the condition that the values from the set \(X\) arrive at the input of the DAT.

In the case of a single digit transformer we shall assume that the system can be in two states with an equal probability: \(x_0\) indicates that the code 0 was inputed at the transformer; \(x_1\) — code 1 was inputed.

Analogously, the system \(Y\) can be found in two states: \(y_0\) — code 0 has arrived at the output of the transformer; \(y_1\) — code 1 has been obtained at the output. Then the conditional entropy

\[
H(Y|X) = -\sum_i r_i \sum_j p(y_j|x_i) \log_2 p(y_j|x_i),
\]

where \(p(y_j|x_i)\) is the probability of obtaining code \(y_j\) under the initial code \(x_i\); \(r_i\) are the probabilities of the states of the initial system \(X\).

In our case the input quantity admits the values 0 and 1 with equal probability and hence \(r_1 = 1/2, i = 1, 2\).

The joint compound system can be in four different states presented in Table 1. It should be noted that \(p(0|0) = p(1|1) = p\), and \(p(0|1) = p(1|0) = s\), since the probabilities of obtaining codes 1 and 0 are the same.

Substituting \(p\) and \(s\) into (1), we arrive at

\[
H(Y|X) = -(s \log_2 p + p \log_2 s) / 2 = -(s \log_2 s + p \log_2 p) / 2 = -(s \log_2 p + p \log_2 s).
\]

Since \(p + s = 1\) expression (2) can be rewritten in two equivalent forms: