MEASURING THE ROOT-MEAN-SQUARE VALUES OF RANDOM VOLTAGES WITH TUBE VOLTMETERS

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Tube voltmeters are widely used in measurement practice, and they possess a high sensitivity which is suitable in many instances for measuring root-mean-square values of noise voltages without preamplification. Their output circuits and pointer indicators have a large time constant, so that the fluctuations at the instruments' output do not exceed their error. These circumstances permit the use of tube voltmeters for measuring the root-mean-square value of noise.

Pointer indicators of modern tube voltmeters consist of moving-coil instruments whose pointer deflections are proportional to the value of the direct current flowing in their coils. Voltmeters intended for ac measurements are provided with linear detectors whose output ac component in proportional, depending on the detector type, either of the amplitude value of the sinusoidal voltage (peak detector), or to the mean value (normal detector). The scale of the instrument, owing to its linearity, is then calibrated, as a rule, either in amplitude of effective values of the sinusoidal voltage.

For sinusoidal voltages the relationship between the amplitude, the mean, and the effective values are represented by the formulas

\[ U_{\text{eff}} = \frac{U_{\text{max}}}{\sqrt{2}}; \quad U_{\text{max}} = \begin{cases} \frac{2}{\pi} U_{\text{max}} & \text{for a full-wave detector} \\ \frac{1}{\pi} U_{\text{max}} & \text{for a half-wave detector} \end{cases} \]

\[ U_{\text{max}} = \begin{cases} \frac{2\sqrt{2}}{\pi} U_{\text{eff}} & \text{full-wave detection} \\ \frac{\sqrt{2}}{\pi} U_{\text{eff}} & \text{half-wave detection}. \end{cases} \]

The coefficients relating \( U_{\text{eff}} \) with \( U_{\text{max}} \) can be used for calibration purposes, and their numerical values in (1) hold for sinusoidal voltages. It follows from the above that in using this type of tube voltmeter, which is intended for working with sinusoidal voltages, it is necessary in measuring the statistical parameters of noise to take the following facts into consideration: 1) what parameter of the input voltage is proportional to the current in the instrument's coil, 2) what parameter of the input voltage is represented by the instrument's calibration.*

For instance, the current in the coil of the LV-9 type voltmeter is proportional to the mean value of the input voltage, whereas their scale is calibrated in effective values.

It should be noted that in the practice of electrical measurements the distribution law of random voltages is often known. Thus, numerous investigations have shown that random voltages due to fluctuation processes in resistors, electron tubes, and semiconductors have a normal distribution law. The widespread nonlinear conversion of

* It is then assumed that the instrument is used in its frequency range.

normal processes by means of linear and square-law detectors results in a Rayleigh or exponential distribution. All the types of the so-called internal equipment noises have known distributions, which have been repeatedly described [1, 2].

The distribution laws of random voltages studied by means of multiseriate measurements are very often known in advance. They include various types of fluctuation of radio and acoustical waves in radio broadcasting and television, fluctuation processes in piezoelectric crystals, in studying thermal radiations, etc. Moreover, in many cases tube voltmeters form an organic part of installations used for measuring intensive fluctuations with known distribution laws.

Let us analyze the work of a tube voltmeter when its input voltage is represented by a stationary random process \( U(t) \) with a one-dimensional probability density distribution function \( f(U) \), assuming moreover that the constant component of the process is equal to zero, since the instrument does not register it.

The mean value of the noise voltage at the output of the instrument's linear detector can be calculated from the formula

\[
\bar{U} = \int_{0}^{\infty} U f(U) \, dU \quad \text{for a half-wave detector,}
\]

\[
\bar{U} = 2 \int_{0}^{\infty} U f(U) \, dU \quad \text{for a full-wave detector.}
\]  

(2)

Let it be necessary to measure a root-mean-square noise value determined from the formula

\[
\sigma = \sqrt{\int_{-\infty}^{\infty} U^2 f(U) \, dU}. 
\]

(3)

For a specific distribution function \( f(U) \) we shall obtain a relationship between \( U \) and \( \sigma \) which can be used for converting the instrument scale.

Let us examine certain characteristic examples of the tube voltmeters' utilization for measuring certain noise parameters.

**Example 1.** An LV-9 tube voltmeter is used. For a sinusoidal voltage we have the relationship

\[
U_{\text{max}} = \frac{\sqrt{2}}{\pi} U_{\text{eff}}. \quad \text{(4)}
\]

Let the voltmeter be fed with a normal random process, for which we find from (2) and (3) that

\[
|\bar{U}| = \frac{1}{\sqrt{\frac{2\pi}{\pi}}} \sigma. \quad \text{(5)}
\]

If the instrument reading is \( U_{\text{eff}} \), then by equating the mean values we can obtain from (4) and (5) the relationship between \( \sigma \) and \( U_{\text{eff}} \)

\[
\sigma = \frac{2}{\sqrt{\frac{\pi}{\pi}}} U_{\text{eff}} = 1.1283 U_{\text{eff}}. \quad \text{(6)}
\]

Thus, if the correction applied for the form factor of the random process is ignored and the instrument readings are taken to represent the root-mean-square value of the input voltage, the measurements will be underrated by approximately 11%.

Let us note that a Gaussian random process has a symmetrical density probability distribution and, therefore, it is possible to ignore the change of the voltage sign at the input of the detector with respect to the voltmeter input.

**Example 2.** Let us now examine the case when a LV-9 type voltmeter is fed with a voltage represented by a random process with an asymmetrical distribution law of instantaneous values, namely, a Rayleigh stationary random process with a probability density function of